ABSTRACT OF THESIS

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Title of Thesis UNSTEADY STATE HEAT TRANSFER TO AND FROM GASES IN LAMINAR FLOW.

This thesis is concerned with the problem of unsteady state heat transfer to and from gases. The advent of the practical gas turbine has revived interest in the regenerative heat exchanger in which the temperatures at any one cross-section are varying with time.

Experiments have been carried out with a circular cross-section passage. Heat transfer takes place within a single vertical tube, around which is an annular guard space. It is believed that this is the first reported work on unsteady state heat transfer to and from gases in laminar flow in a circular tube. Because of the magnitude of heat losses at lower values, the investigation is confined to Reynolds numbers between 1,000 and 2,500.

Within this range it is shown that at moderate temperature differences, the processes of heating and cooling of gases reveal similar heat transfer properties. When the tube wall temperature is either constant or has a uniform temperature gradient, it is shown that the Nusselt number is in agreement with that predicted for steady state heat transfer for the same wall temperature distribution.

The investigation deals with single cycle operation, referred to as "single blow", in which the supply is either hot or cold, and with cyclic operation as a parallel-flow regenerator. Heat transfer coefficients obtained from the cyclic operation tests, agree with those from the "single blow" tests.

An analytical expression for the prediction of outlet temperatures, assuming a varying supply inlet temperature and

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UNSTEADY STATE HEAT TRANSFER TO AND FROM GASES IN LAMINAR FLOW.

by

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Thesis submitted for the degree of Doctor of Philosophy at the University of Edinburgh.

May, 1954.

CONTENTS.

NOTE.

In this thesis, for simplification, the following convention of Schack and others will be used:

"gas" refers to the hot supply fluid,

and "air" to the cold supply fluid.

NOMENCLATURE.

A	cross-sectional area normal to gas flow.
c_p	specific thermal capacity of gas at constant pressure.
d	diameter of circular tube.
G	rate of flow per unit cross-sectional area.
g	acceleration due to gravity.
h	heat transfer coefficient.
K_{eb}	kinetic energy velocity distribution coefficient.
k	thermal conductivity of gas.
k_T	thermal conductivity of tube material.
L	test section length.
M	water equivalent of tube per unit length.
m	rate of flow.
P	wetted perimeter of tube.
p	pressure.
Δp	pressure drop.
Q	quantity of heat
r	radius.
s	radius of tube.
v	velocity of gas stream
W	water equivalent of gas flowing per unit time.
x	distance of point in gas path from entry.

z time elapsed from changeover.

$\theta(x,z)$ instantaneous gas temperature.

$\tau(x,z)$ instantaneous tube temperature.

μ viscosity.

ν kinematic viscosity.

ρ density.

σ contraction area ratio.

ϕ pressure drop coefficient.

ς corrected time.

Subscripts.

am arithmetic mean.

i at $z = 0$

l local

M mean

o tube centre line, $r = 0$

s tube surface, $r = s$.

Dimensionless Numbers and Parameters.

G_z Graetz number $\left(G_z = \frac{\pi}{4} R_e . P_r \frac{d}{L} = \frac{m C_p}{k L}\right)$

G_r Grashof number $\left(G_r = \frac{g d^3}{T_s \nu^2}[T_s - \theta_o]\right)$

N_u Nusselt number $\left(N_u = \frac{hd}{k}\right)$

P_r Prandtl number $\left(P_r = \frac{c_p \mu}{k}\right)$

R_e Reynolds number $\left(R_e = \frac{Gd}{\mu}\right)$

ξ, Λ Non-dimensional length $\left(\xi = \frac{hPx}{W}, \Lambda = \frac{hPL}{W}\right)$

η Non-dimensional time $\left(\eta = \frac{hPz}{M}\right)$

SYNOPSIS.

This thesis is concerned with the problem of unsteady state heat transfer to and from gases. The advent of the practical gas turbine has revived interest in the regenerative heat exchanger in which the temperatures at any one cross-section are varying with time.

Experiments have been carried out with a circular cross-section passage. Heat transfer takes place within a single vertical tube, around which is an annular guard space. It is believed that this is the first reported work on unsteady state heat transfer to and from gases in laminar flow in a circular tube. Because of the magnitude of heat losses at lower values, the investigation is confined to Reynolds numbers between 1,000 and 2,500.

Within this range it is shown that at moderate temperature differences, the processes of heating and cooling of gases reveal similar heat transfer properties. When the tube wall temperature is either constant or has a uniform temperature gradient, it is shown that the Nusselt number is in agreement with that predicted for steady state heat transfer for the same wall temperature distribution.

The investigation deals with single cycle operation, referred to as "single blow", in which the supply is

either hot or cold, and with cyclic operation as a parallel flow regenerator. Heat transfer coefficients obtained from the cyclic operation tests agree with those from the "single-blow" tests.

An analytical expression for the prediction of outlet temperatures, assuming a varying supply inlet temperature and non-uniform initial wall temperature, is developed, and this is used in the calculation of the "single-blow" results.

CHAPTER I

INTRODUCTION

THE PRACTICAL IMPORTANCE OF THE STUDY OF UNSTEADY STATE HEAT TRANSFER.

In the regenerative heat exchanger we have a practical application of unsteady state heat transfer. Here a certain amount of heat is available in a tube wall or matrix, and this heat is transferred to air, which enters the tube with a temperature which is constant, or nearly so, with respect to time. Therefore, as the air, initially cold, passes through the tube, the air will be heated and the tube will be cooled. The temperature of both substances at any one point will vary continuously with time.

For the steady state case the heat transfer coefficients have been calculated theoretically previously. The comparison of these values, and those obtained experimentally for the unsteady state case, is one of the objects of this research.

In the regenerator there are two distinct operations, one in which the tube is heated by a supply of hot gas which is itself cooled, and the succeeding period in which the tube is cooled, due to the passage of cold air, which is heated. These two distinct phases may not at first sight be expected to possess similar heat transfer properties. It is not possible to produce unique theoretical values of heat transfer coefficients for the unsteady state case.

Some experimental values have been obtained by COX, JOHNSON and others for square, triangular and other non-circular cross-section passages, but as the circular cross-section is the elementary passage in heat transfer convection work, it was considered advisable to investigate this case.

The previous related investigations are summarised and discussed at the end of this thesis.

<u>Order of Presentation.</u>

In the transfer of heat by convection, investigations have been generally confined to the steady state case. In view of the overwhelming concentration on this aspect of the subject, those factors influencing heat transfer to gases in unsteady state laminar flow are stated in the first pages of this thesis.

As agreement is found between experimental results and predictions for steady state heat transfer under conditions of similar wall temperature distribution, the relevant theories for steady state operation are summarised.

The form of apparatus, which results from the choice of a single circular tube as the heat transfer section, is described and illustrated.

The effect of heat transfer at any cross-section is to alter the temperature of the gas flowing through it, and also the temperature of the tube wall in contact with it. This process is analysed for the case where the supply inlet temperature varies with time and the tube initial temperature is not uniform.

Experiments were carried out with single blow operation, either hot or cold supply, and with cyclic operation of the apparatus as a parallel-flow regenerator. In both cases the pressure drop over the vertical test section is taken as the guide to the desired flow condition, i.e. laminar flow. This is shown to exist within the inner tube, even for $R_e > 2,500$.

From the theoretical analysis, values of the gas outlet temperature can be determined for the particular supply inlet and initial tube temperature boundary conditions. A comparison between an experimental curve corrected for heat losses, and curves obtained from the calculated values is made, and the heat transfer coefficient h_l determined.

In cyclic operation, the method of obtaining the experimental value of regenerator efficiency is discussed. This value is compared with one from a formula given by HAUSEN, and in this way the heat transfer coefficients are obtained.

In unsteady state heat transfer, the amount of theoretical work on the prediction of outlet temperatures for various boundary conditions is very much greater than the amount of experimental work. Those investigations which are related to the contents of this thesis are considered before making any conclusions.

Experimental results.

The only available standard of comparison for these results with a circular cross-section, are the theoretical values for steady state heat transfer. It is not possible to derive a theoretical expression for unsteady state heat transfer, other than in a non-dimensional form, because of the complex boundary conditions.

When the wall temperature is uniform throughout, it is shown that the values of heat transfer coefficient obtained from experiment are of the order predicted by the theories of steady state transfer, and that they are similar for either heating or cooling of the tube. The lower values derived from some of the cooling tests are traced to a reversal of the temperature profile within the passage length.

When the tube wall temperature gradient is approximately uniform, and providing that the processes of heating and cooling of the wall do not take place at the same time, the experimental heat transfer coefficients are found to exceed those predicted for steady state, having a similar wall temperature distribution. It is submitted that this difference is due to the increased heat transfer coefficient which occurs in the entry length.

CHAPTER II

PRINCIPLES OF HEAT TRANSFER RELEVANT TO GASES IN UNSTEADY STATE CONDITIONS.

The principles of heat transfer relevant to gases in unsteady state conditions are considered in two sections.

The terminology of heat transfer by convection is based on the assumption that this is taking place under steady state conditions. The fact that the majority of heat systems seldom operate under continuously steady operation is generally neglected because the deviation is negligible for practical purposes.

Therefore in the first section, the factors influencing heat transfer in the unsteady state case have been considered briefly. Because of the moderate temperature differences of this investigation, two assumptions have been made for the unsteady state case,

(i) the entry length, $L_e = 0.05\, d. R_e P_r$

(ii) the velocity profile beyond the entry length is parabolic.

The values of Nusselt number determined in the unsteady state experiments are to be compared with those theoretically predicted for the steady state case. A summary of these theories is given in the second section.

Distinction is made between the values of N_u based on local mean temperature difference and arithmetic mean temperature difference.

In the steady state case, the local mean temperature difference is of importance when the tube wall temperature gradient is uniform, the arithmetic mean temperature difference is important when the tube wall temperature is constant.

In these theories, the physical properties are assumed constant across the cross-section. It is known that some physical properties vary with temperature, and therefore for large temperature differences, the assumptions of Graetz, etc., are not quite correct.

1. FACTORS INFLUENCING HEAT TRANSFER TO AND FROM GASES IN UNSTEADY STATE.

Introduction

When a fluid flows through a straight tube at a sufficiently small velocity, the particles of fluid move in an axial direction only. This is referred to as laminar flow.

When this velocity is increased above a certain value, this axial flow is combined with a flow of the particles normal to the axial direction, in which case mixing of the fluid layer occurs. This state is referred to as turbulent flow.

Between these two distinct states there is a transition region wherein the phenomena cannot be so strictly defined.

In the laminar flow region, to which this investigation is restricted, heat transfer within the fluid is due mainly to conduction.

It is also necessary to distinguish between free and forced convection; in the former case, fluid flow arises from the variation in density caused by the heat transfer, whereas in the latter case fluid flow results from a pressure difference applied across the tube ends.

The simplest cases of laminar flow are the isothermal or steady state ones, in which the temperature at any one point in the tube wall or gas remains constant with respect to time. e.g. Gas supplied at constant temperature to a tube whose walls are maintained at their initial temperature distribution by the supply of heat from an external source.

In the non-isothermal or unsteady state case, the temperatures at any one point vary with time because of the transfer of heat, e.g. Gas supplied at constant temperature to a tube at a different temperature and isolated from any heat source or sink.

Heat Transfer Coefficient.

The heat transfer coefficient h can be defined by the relation,

$$q = hA.\Delta\theta \qquad (2.1)$$

where q is the amount of heat transferred in unit time at the solid-fluid interface of area A , where the representative temperature difference is $\Delta\theta$.

It must be remembered that the coefficient will have a different value depending on the particular basis of temperature difference used. (NORRIS & STREID).

In the analysis included here two coefficients are considered.

The local coefficient h_l is based on the local temperature difference existing at any one position. At the same time, however, it has been necessary for the purposes of analysis to assume that h_l is constant with respect to position and time. The value of h_l obtained from the experiments does vary with time.

An arithmetical mean temperature difference has also been adopted (See Appendix VI) similar to that used in steady state analysis. Based on this, the value of the coefficients h_{am} have been calculated for all the results obtained.

From these heat transfer coefficients the corresponding Nusselt numbers, Nu_l and Nu_{am}, are obtained where,

$$Nu = \frac{hd}{k} \quad .$$

<u>Velocity profile.</u>

In isothermal laminar flow in a circular pipe at a cross-section a great distance from the entrance, the velocity profile is parabolic. When heat transfer takes place the velocity profile is distorted because the viscosity is no longer constant across the cross-section. It can be shown by approximation however, that in the case of a temperature difference between wall and tube centre line of 100°F, the velocity profile determined, taking the changes of viscosity into account, departs very little from the parabola of the isothermal case.

Where the fluid flow is unsteady other modifications may be expected, but in the case of constant mass flow of gas through a uniform passage, these will depend on the rate of axial temperature change.

In view of the above calculation and the small rates of axial temperature change, it is assumed that in the following investigation, in the fully developed flow region, the velocity profile is parabolic, both in steady state and unsteady cases.

Temperature Profile.

In isothermal laminar flow, the temperature is, of course, uniform at all points in any cross-section. In non-isothermal steady state it has been found previously by calculation and experiment that the temperature profile resembles the velocity profile but is not coincident with it.

In the unsteady state case under examination, it will be seen that at exit the temperature profile differs little from the steady state case except under certain conditions.

Entry Length.

In steady state heat transfer to a tube, it is known that entry and for a distance downstream, the velocity profile is not fully developed to its parabolic form, and that the local heat transfer coefficients within this region are higher than those obtaining further downstream, in the region of fully-developed laminar flow. This will also apply in unsteady state cases, but whereas it is comparatively easy to avoid considering the entry length in steady state experiments, it is very much more difficult to do this in the unsteady state cases. In this particular investigation, the entry length is as important a part of the heat transfer surface as that in the fully-developed region and must be included.

It is appreciated that consideration of the actual conditions in the entry length must be attempted at some time in a programme of work of this kind.

In steady state, fully-developed flow is considered to exist only at a distance

$$L_e = 0.05\, d .\, R_e P_r \quad , \qquad (2.2)$$

beyond the entry to the passage, where d is the tube diameter.

It is considered that in this first instance, this relationship can be applied to the unsteady state and therefore in this investigation, the entry length varies between approximately 25% and 60% of the total tube length.

The Effect of Temperature on Physical Properties.

Certain physical properties of fluids which influence the heat transfer process, are dependent on temperature. If, therefore, the reference temperature is changing, these properties must also change, and the heat transfer process accordingly. At high temperature differences, this becomes particularly important. The properties concerned are density, viscosity and thermal conductivity.

Density. In the particular experimental arrangement adopted, i.e. a vertical tube, it was to be expected that the changes of density resulting from the heat transfer might possibly encourage free convection, particularly at low Reynolds numbers. It was therefore considered advisable to work in the upper region of laminar flow, and in fact, at $R_e > 1000$.

(The effect of heat losses in the apparatus also increases with decrease in flow rate).

Viscosity. When a gas is heated, its viscosity increases. If there is a considerable temperature variation in the gas, there is a corresponding variation in viscosity, which should be taken into account. This change of viscosity across the flow cross-section distorts the velocity profile and modifies the heat transfer coefficient. In the case of steady state heat transfer to fluids such as air and water, the alteration of h with viscosity has been shown to be proportional to the ratio $\frac{\mu_m}{\mu_s}$. In this experimental work the temperature distribution did not vary widely and this effect could not be gauged. At high temperature differences, however, this may account for any difference between heating and cooling runs.

Thermal Conductivity. This property also varies with temperature and has an influence on the heat transfer process. It has however been considered constant during any one test in this investigation.

2. <u>THEORIES OF HEAT TRANSFER TO A FLUID IN LAMINAR FLOW IN A CIRCULAR TUBE UNDER STEADY STATE CONDITIONS.</u>

The heat transferred within a circular tube to a fluid in laminar flow under steady state conditions, depends on the tube wall temperature distribution.

<u>Uniform Wall Temperature</u>

The problem has been solved by Graetz and Nusselt for the case of the wall at <u>uniform temperature</u> and the fluid entering at some different temperature, assuming the Poiseuille velocity distribution (GOLDSTEIN). The variation in thermal properties of the fluid with temperature is neglected.

If the wall temperature is $\tau = 0$, and the gas inlet temperature is $\theta_i = 1$ (i.e. uniform across the cross section), then θ_M the mixed mean temperature at any cross section is given by

$$\theta_M = 0.819e^{-14.627X} + 0.0976e^{-89.22X} + 0.01896e^{-212X} + \dots \qquad (2.3)$$

$$\text{where } X = \frac{kx}{v_M d^2} = \frac{L/d}{R_e . P_r} .$$

From this can be determined the heat transfer to the wall in unit time for a given length.

The Nusselt number, based on a mean temperature difference, which is taken to be the arithmetic mean of the temperature differences between the fluid and the wall at entry and exit, is

$$Nu_{am} = \frac{R_e . P_r}{L/d} . \left(\frac{1-\theta_M}{1+\theta_M} \right) \qquad (2.4)$$

where, from eqn. (2.3) $\theta_M = f\left(\frac{L/d}{R_e . P_r} \right)$.

Nu_{am} values may therefore be plotted against $\frac{R_e P_r}{L/d}$ or the Graetz number, $G_z = \frac{\pi}{4} R_e P_r . \frac{d}{L}$.

The local Nusselt number derived from the same theory is found to vary; at $x = 0$, $Nu_l = \infty$, and as $x \to \infty$, it approaches the limit; $Nu_l = 3.65$. (2.5)

In steady state for these boundary conditions, Nu_{am} is the most suitable value, although Nu_l helps to define the limit of entry length.

For values of $G_z < 10$, to which the present investigation is restricted, I consider Nu_{am} for steady state to be given by the Graetz equation (2.4), which has an asymptote,

$$Nu_{am} = \frac{2}{\pi} Gz \quad . \tag{2.6}$$

This upper limit applies when the gas temperature at outlet equals the wall temperature.

Free Convection and Laminar Flow.

The effect of free convection on a fluid flowing in laminar flow through a vertical tube was considered by MARTINELLI et al. The tube had uniform wall temperature. They predicted values of Nu_{am} as a function of the Graetz number, with the dimensionless product $\left(\frac{G_r P_r d}{L}\right)_s$ as a parameter.

The predicted values are shown in Fig. 1.

$Gr_s = \frac{g d^3}{T_s \nu^2} (\tau_s - \theta_o) =$ Grashof number, for a perfect gas, based on the temperature difference $(\tau_s - \theta_o)$. All properties are evaluated at the wall temperature τ_s, T_s is the value in absolute units.

Uniform Temperature Gradient along Wall.

In the case of a circular tube having walls maintained at a <u>uniform temperature gradient</u>, and where the fluid has the Poiseuille velocity distribution, it is shown by GOLDSTEIN, that, based on the fluid mixed mean temperature, the value of the local Nusselt number at any cross-section is

$$Nu_l = \frac{48}{11} . \qquad (2.7)$$

It can be seen by inspection that under these conditions

$$Nu_{am} = \frac{48}{11} . \qquad (2.8)$$

CHAPTER III

EXPERIMENTAL APPARATUS

When this problem of unsteady state heat transfer was first considered, it was decided that in the case of the study of flow through a regular passage, the basic cross-section should be a circular one. Further consideration suggested that if the heat transfer process differed in heating and cooling runs, it would be possible to investigate the difference if a large diameter passage was used. Other factors, e.g. ratio $\frac{\text{length}}{\text{Diameter}}$, ratio $\frac{\text{thermal capacity of tube wall}}{\text{thermal capacity of gas}}$, thermal conductivity of metal, and Reynolds number, all influenced the final choice of test section, viz., $\frac{1}{2}$ inch bore and 72 inches long.

An outer tube of $1\frac{1}{8}$ inch bore is arranged around this test section so that between these two lies an annular cross section with the same hydraulic diameter of one half inch, as the inner test section. The same supply fluid passes through this annulus as through the test section and the ratios $\frac{\text{thermal capacity of tube wall}}{\text{thermal capacity of gas}}$ are the same for each flow passage. The assumption is made that there is no radial heat flow at the mean diameter of the inner tube wall.

After these decisions had been made, the general principle adopted in devising the apparatus was, that it should be possible to measure the effect, on a single circular vertical tube, of both "single-blow" and cyclic operation. A comparison between the results from these two methods would test the belief (HEAT TRANSFER CONFERENCE, 1951) that heat transfer coefficients, obtained in one case, could not be applied to the other.

Experimental simplification recommended the parallel flow arrangement which was finally adopted, although, generally, regenerators operate in contra-flow. The main difference, the resulting wall temperature distribution, is referred to later. At the same time, it is noted that HAUSEN shows that under controlled conditions, theoretical regenerator efficiencies of 70 and 80% can be obtained in parallel flow operation, whereas the parallel flow recuperative heat exchanger is restricted to a maximum efficiency or thermal ratio of 50%.

The general arrangement of the apparatus is shown in Fig. 2.

The Heat Transfer Section.

Air flowing along the inner and outer walls of a circular tube, will either receive heat from the wall or supply heat to it. There is, of course, a difference in the process of transferring heat to the inner wall of an annular cross-section and to the inner surface of a circular cross-section. Nevertheless if the air temperature is the same inside and outside the tube, we approach the condition of no heat flow at the tube mean diameter, and approximate to a true storage of heat.

It is therefore assumed that the heat transferred to the wall of the inner circular passage heats up a thickness of tube equal to half the thickness of the actual tube. See. Fig. 3.

The inner tube is suspended from a flange at the top and is free to expand downwards. It is maintained concentric with the outer tube by Steatite pins which are screwed in bosses on the outer tube.

The outer tube is fixed at the top while its lower end is free to slide in an expansion gland. The outer tube was aligned vertically, whilst the central positioning of the inner tube was carried out by measurement. For initial adjustment, complete freedom of relative movement is possible at the suspension flanges of both tubes.

The material selected for the solid drawn tubes was Tough Pitch High Conductivity Copper, which had an excellent finish in the "as drawn" condition. The physical properties of this material are:

density	0.323 lbs/cu.in.
Specific thermal capacity	0.095 B.Th.U./lb. ^{o}F.

thermal conductivity 5.25×10^{-3} B.Th.U/in.sec.^{o}F.

The conditions which the apparatus must satisfy approximately are (a) infinitely high thermal conductivity normal to the direction of gas flow,

(b) negligible thermal conductivity parallel to the direction of gas flow.

Copper was considered as the most suitable, practical material.

Air Supply and Selection Arrangement (See Fig. 4).

Separate supplies of hot air ("gas") and cold air ("air") were required and two centrifugal blowers, a "gas blower" and an "air blower" were fitted. A variable resistance in series with each motor was used to regulate the air mass delivered by each. Air from the gas blower passed over two separate electric resistances, the current to which passed through variable resistances.

Each supply line had a separate orifice meter which had been calibrated against the discharge from a standard gas holder. Even with the normal variations in supply current, it was found that the mass flow and temperature of the hot air could be maintained sensibly constant.

Before reaching the valve box, the fluid passed through guides to straighten the flow and stabilise the temperature across the cross-section.

Since either hot or cold supply was required at any instant, a selector valve box as shown was made. Several advantages of the finished form are given.

1. Minimum deviation of the gas flow direction.
2. Sudden cut-off of the unwanted gas or air supply and its discharge to atmosphere.
3. The momentum of the fluid at changeover helps to obtain this cut-off.
4. Minimum heat capacity of the valve box.

This point 4 arises from the fact that some faces of the valve box are in contact with the supply, which is passing to the test section, and therefore if such a surface has a temperature differing at any instant from that of the supply, there will be a variation in supply temperature at the entrance to the test section.

The effectiveness of the valve box, with regard to point 1, is shown in the pressure drop results, and to point 4, in the record of inlet temperature readings during any of the cyclic tests.

The problem of sealing between high and low pressure supply streams which arises in practical regenerative heat exchangers does not arise here, because both supplies are practically at the same pressure, and that is only a few inches of water above atmospheric. Any slight leakage at the valves in this experimental arrangement has no influence, other than when the size of the exit orifice is changed.

Fig. 5 is a photograph of the valve box in the assembled apparatus.

Pressure Control.

One practical difficulty which required attention in the design stage, was the control of pressures in the various parts of the apparatus.

e.g. When 'gas' is being supplied to the test section, the flow resistance is due to the length of test section, the exit pipe and exit orifice. If now the supply is changed, and the 'gas' discharged directly to the atmosphere at the selector valve, the flow resistance will decrease, the 'gas' mass flow will increase, and its temperature at exit from the heater will fall. A return to hot supply at the test section will reverse this procedure and consequently there will be a fluctuation in supply temperature.

To overcome this, weighted flap valves were fitted at the exhaust branches of the selector valve box, and these supplied the necessary resistance to flow during exhausting.

Comparison of the flow conditions existing in the inner tube and the annulus show that it is necessary to supply an additional resistance at outlet from the annulus which will correspond to the flow resistance of the exit pipe and orifice. Initially this was supplied by a spring-controlled throttle device, but when an alteration was made in the method of measuring the outlet temperature from the inner tube, this device was replaced by that shown in Fig. 3. Any adjustment to the resistance was made by plugging up or unstopping some of the thirty or more outlet holes.

Temperature Measurement.

At all times, unsteady state heat transfer is taking place and thus temperatures are varying continuously with time and position. Indications of these temperatures could only be given by thermocouples, which were made of iron-constanton glass fibre insulated wire. Where the gas temperatures were steady or where the temperatures were those of the tube material itself, 24 B. & S. gauge wire was used. For the gas temperatures which varied rapidly, i.e. test section inlet and outlet, 30 B. & S. gauge wire was used. In all cases the junction was made by welding with borax flux.

The thermocouples for measuring temperatures at the mean diameter of the inner tube wall were held in firm contact with the metal at this point. There were two of these couples at a distance of one foot from either end of the test section.

The couple, indicating the temperature at inlet to the inner tube, had its junction in the plane of the cross-section at entry, the wires themselves leading upstream on the centre line for a distance of two inches, before leading out through the side wall. It was assumed that, due to entry conditions, the temperature across this cross-section would be uniform.

At exit from the test section however, the profile would no longer be linear and the banjo arrangement suggested by H. Cohen was used. Three thermocouples with their junctions at different distances from the centre line were arranged in a distance piece of "Tufnol" as shown in Fig. 3. Assuming a parabolic velocity profile at exit, the mixed mean temperature could be determined from the values indicated by these couples.

Physical mixing of the gases at outlet was not considered possible because of the effect of the heat capacity of the mixing chamber walls. Traversing of a single couple across the flow at outlet was precluded by the variation in temperature with time.

All these varying temperatures were recorded by a 12-point Electronik potentiometer recorder which, with a range of 0-300°F, carried out a complete sequence of twelve readings within a period of one third of a minute. In fact only eleven couples were attached to the instrument and the temperature of the supply at inlet to the test section was recorded at twice the frequency of the others.

Exit Flow Measurement.

The air or gas from the test section is finally passed through a flat plate orifice situated some distance downstream of the outlet. The pressure drop across this orifice was registered on an inclined tube manometer. The actual orifice plates used were standardised by discharging air, from a standard five cubic foot gas holder, through the orifices, when fitted in the actual exit piping.

During the experiments the flow through the test section was found to fluctuate no more than a total of 3%, and there was no marked variation in gas temperature at the exit orifice.

Pressure Drop Measurement.

The pressure drop over the test section was measured by a Casella micrometer reading manometer. Tappings were made upstream of the entrance a distance of two inches, i.e. in the body of the control valve, and downstream of the exit a distance of one inch.

The zero setting of the instrument is made with no flow through the test section, the air temperature and pressure within the section being noted. Pressure drop measurements can then be made throughout the test.

As the tapping points are approximately six feet vertically above each other, it will be seen that, if the density of the air within the tube alters, the zero reading is affected. Therefore when pressure drop readings have been taken in non-isothermal conditions, these readings are corrected for this variation in density.

Insulation.

Any insulation applied to the outside of the outer tube represents an additional heat capacity and for this reason the type of insulation used must have minimum thermal capacity and conductivity.

In the final arrangement adopted the outer surface of the outer tube was polished and then wrapped with one layer of half-inch thick low-density Fibreglass insulation.

Surface Conditions.

Care was taken to avoid damage to the surface of the solid drawn copper tube test section. Continuous passage of heated air across this surface results in the formation of a very thin oxide film which is easily removed, but to ensure that the effect of this is the same in all experiments quoted here, none of these were made until this film was established.

The entry end of the inner tube is radiused in order to reduce turbulence at this position.

CHAPTER IV

TEMPERATURE VARIATION OF FLUID AT OUTLET FROM TEST SECTION.

The purpose behind the experiments detailed in this investigation is the determination of heat transfer coefficients. It will be shown in Chapter V how this is done by comparing experimental and calculated values of the gas outlet temperatures from the test section. In this chapter the method in which these different sets of values are obtained is considered.

The experimental temperature records provide readings of outlet temperatures at three points at different radii across the cross-section. A calculation, in which the velocity profile is assumed parabolic, using a temperature profile derived from the above three temperature values, determines the radial position for reading the mixed mean temperature of the gas at outlet i.e. at $r/s = 0.56$.

There is a heat loss in the experiments and after considering the reasons for this, a method of correcting the temperature values for this effect is indicated.

In the theoretical prediction of the gas outlet temperature, the major assumption is that the thermal conductivity of the tube material is negligible in a direction parallel to that of the gas flow and infinite in a direction at right angles to that of the gas. To the resulting equations for heat transfer between the gas and the wall, are applied the following boundary conditions,

(i) supply inlet temperature varies with time, i.e.

$$\theta = \theta_0(z) \quad \text{at} \quad x = 0$$

(ii) inlet tube temperature need not be uniform, i.e.

$$\tau = \tau_0(x) \quad \text{at} \quad z = 0.$$

From the resulting equations, an analytical expression for gas temperature at any point in the passage length is obtained, and therefore an expression for the gas outlet temperature. An expression for the wall temperature at any point is determined by symmetry of the equations.

1. SINGLE BLOW EXPERIMENTS

Description of a Single Blow Test.

When the micromanometer is zeroed, the blowers are started and adjusted to give a steady supply. The necessary adjustment is made at the weighted flap valves to ensure steady flow at changeover to the other supply. A preliminary isothermal pressure drop check is then carried out with the appropriate supply.

At this stage, in the case of a heating run or "gas blow" the gas supply will be exhausting to atmosphere at the valve box. The heater is now switched on. When the gas temperature is steady at its desired value, as indicated by the recorder, a final check is made of all the orifice readings, i.e. pressure difference and pressure, before changeover to the hot supply.

Time is measured from the valve changeover and at intervals of one minute from then, orifice flow readings are noted.

It has been found that from the point of view of heat losses the maximum useful duration for such a test is about ten minutes. During that time, two or three pressure drop readings are generally made.

In the case of a cooling run, or "air blow", the test section is first heated to steady temperature conditions. The period of the air blow begins at the moment of changeover to the cold supply, the gas being exhausted to atmosphere.

Temperature Records.

An actual temperature record for a gas blow is shown on Fig. 6, whilst Figs. 7 and 8 are copies of air blow records.

Supply Inlet Temperature. In each of these different cases it is seen that the supply inlet temperature is not constant throughout the blow period, although it does approximate to a constant value with the increase of time. It is mainly because of this, that the general form of supply inlet temperature of eqn. (4.5) is adopted in the theoretical analysis.

Tube Initial Temperature Distribution. If it is remembered that the tube temperature is measured at two positions only, but that it is assumed that the tube temperature distribution is linear at all times, it will be seen from the above records that the tube initial temperature varies in different tests. In the hot supply tests as in Fig. 6, the temperature can be assumed uniform along the tube, but this is certainly not the case in the air blow tests. This variation explains the adoption of the general boundary condition, eqn. (4.6) for the tube initial temperature.

Gas Outlet Temperature. At outlet from the test section, the gas temperature is indicated by three thermocouples arranged at values of r/s of 0, 0.4, and 0.8. In the calculation of results, the outlet temperature required is the mixed mean temperature. The radius at which this temperature may be said to exist was determined from several experimental temperature profiles, assuming a parabolic velocity distribution.

This mixed mean temperature may be expressed as

$$\theta_M = \frac{2}{s^2 v_M} \int_{r=0}^{r=s} v\theta_r \, . \, dr \qquad (4.1)$$

and in the above case, the velocity at radius r, is

$$v = v_0 \left(1 - \frac{r^2}{s^2}\right) \quad .$$

The radius at which the actual temperature and the mixed mean temperature have the same values, was found to be given by the ratio ,

$$r/s = 0.56 , \qquad \text{where} \quad s = d/2 .$$

With the flow cross-section dimension, s = 0.25 ins.,
this radius will be 0.14 ins.

Therefore, in the experimental records, the mixed mean temperature at outlet is obtained by plotting the outlet temperature profile and reading off the value of r = 0.14 ins.

The r/s value of 0.56, agrees well with Crocco's calculation (ECKERT) for reference temperature,
viz.,

$$\theta_{ref} = \theta_o + 0.58(\theta_s - \theta_o) .$$

Heat Loss to Insulation.

In the early stages of the experiments, whilst a choice of suitable insulation was being made, an estimate of the heat loss at various positions on the outside of the outer tube was carried out using Elliott heat flow meters. In gas blows, as expected, these showed the greatest heat loss to be at the lower end.

An estimate of the rate of heat loss to the insulation, as a proportion of the rate of transfer of heat to or from the totalweight of gas flowing through the inner and annular cross sections, was made for heating and cooling blows. The results obtained were:

gas blow - % heat loss to insulation = 5%

air blow - % heat loss to insulation = 0 .

Conduction along the Tube parallel to Flow Direction.

In the theory, thermal conductivity is considered negligible along the tube, and it is necessary therefore to examine the magnitude of the heat flow in this direction over the actual test length.

The following estimation is due to TIPLER.

Cross-sectional area of tube in direction of gas flow $= a$.

$$\left.\begin{array}{l}\text{Temperature gradient along tube, given}\\ \text{approximately by the fluid temperatures}\end{array}\right\} \simeq \frac{\theta_{am_o} - \theta_i}{L}$$

$\therefore$ Instantaneous rate of heat flow $= \dfrac{k_T . a\,(\theta_{am_o} - \theta_i)}{L}$ (4.2)

In the period time z, the heat given up by the gas to the tube is of amount, $Wz(\theta_{am_o} - \theta_i)$.

$\therefore$ Mean rate of heat transfer $= W(\theta_{am_o} - \theta_i)$ (4.3)

$$\therefore \frac{\text{Rate of longitudinal heat conduction}}{\text{Rate of heat transfer, gas to tube}} = \frac{k_T\, a}{WL} \qquad (4.4)$$

In this investigation, $k_T = 5.25 \times 10^{-3}$ B.Th.U/in.sec.$^{\circ}$F.

$a = 0.0535$ sq. ins.

$W = 0.000189$ B.Th.U./sec.$^{\circ}$F.
(at $R_e = 1840$)

$L = 72$ ins.

$$\therefore \frac{k_T\, a}{WL} \simeq 0.02 .$$

$\therefore$ The effect of longitudinal conduction is fairly small and it has been neglected throughout.

Heat Transfer in Annular and Circular Passages.

Although the equivalent diameters and the ratios

$\dfrac{\text{cross-sectional area of tube material}}{\text{cross-sectional area of passage}}$ are the same for both passages, this does not ensure exactly similar temperature conditions in both the annulus and inner tube. e.g. The supporting pins for the inner tube pass through the annular flow area and may promote turbulence there.

In the preliminary stages of setting up the apparatus, readings were taken of the temperature at exit from the inner tube and the annulus, and it was considered that the effect of the difference of temperature, would be effectively taken care of by the heat loss correction outlined below.

Heat Loss Correction (See Appendix I)

None of the above factors, heat loss to insulation, conduction along tube wall, and difference of annulus and tube heat transfer properties, have been taken into account in the theoretical analysis and this will ofcourse influence the results obtained. It is, however, possible to correct the experimental results.

Consider a heating cycle - for any period from the start of a gas blow, the heat given up by the gases is known. For the same period, the heat retained in the tube wall can be estimated from the tube temperatures. If it is assumed that the heat retained in the tube is the actual heat quantity exchanged, then the period during which the gas has, in fact, given up this amount of heat can be determined from the tube temperatures. The time scale is then altered accordingly. (For this correction, it is assumed that the gas inlet temperature is constant, and that the gas outlet temperature varies directly with time). This modifies the results progressively with time; it does not of course affect the value at $z = 0$.

The heat loss correction actually used was further simplified as shown in the appendix.

Using this method, the outlet temperature of Fig. 9 is modified to the form shown in Fig. 10.

2. THEORETICAL ANALYSIS.

General Solution

In Appendix II, analytical expressions are developed, which make possible the prediction of gas and wall temperatures, at any cross-section and any time, in this unsteady state heat transfer process.

The following assumptions are made,

(i) The thermal conductivity of the tube material is zero in a direction parallel to that of the gas flow and infinite in a direction normal to that of the gas flow.

(ii) The thermal properties of the gas do not vary during the process, nor does the heat transfer coefficient.

(iii) At any instant the temperature of the gas across a cross-section of the passage is constant at a mixed mean value.

(iv) There is no external heat loss.

In this analysis the non-dimensional groups, introduced by others,

$$\xi = \frac{hPx}{W}, \quad \text{and} \quad \Lambda = \frac{hPL}{W} \quad \text{for length,}$$

and
$$\eta = \frac{hPz}{M} \quad \text{for time,}$$

are used. These groups are expressions of the ratio of two heat quantities.

P is the wetted perimeter of the tube; L, its total length; W, the water equivalent of gas flowing per unit time; and M is the water equivalent of the tube per unit length. (In this investigation M applies to the thickness of tube between its inner and its mean diameter).

The general boundary conditions of supply inlet temperature which varies with time, and tube temperature which need not be constant, are expressed as,

$$\theta = \theta_0(\eta) \quad \text{at} \quad \xi = 0 \tag{4.5}$$

and
$$\tau = \tau_0(\xi) \quad \text{at} \quad \eta = 0 \tag{4.6}$$

respectively.

From eqn. (A2.15) the gas temperature at outlet, $\xi = \Lambda$, is given by,

$$\theta(\Lambda, \eta) = e^{-\Lambda-\eta}\Big[\theta_0(\eta)e^{\eta} + \sqrt{\Lambda}\int_0^{\eta} \frac{\theta_0(\alpha)e^{\alpha}}{\sqrt{\eta-\alpha}} \cdot I_1\left\{2\sqrt{\Lambda(\eta-\alpha)}\right\} d\alpha$$

$$+ \int_0^{\Lambda} \tau_0(\alpha)\, e^{\alpha}\, I_0\left\{2\sqrt{\eta(\Lambda-\alpha)}\right\} d\alpha\Big] \tag{4.7}$$

where $I_0(x)$ and $I_1(x)$ are modified Bessel functions of the first kind, of zero, and first order respectively.

Eqn. (A2.16) gives a similar type of expression for the tube wall temperature, but as this has not been used directly in this investigation it is not reproduced here.

Eqn. (4.7) reduces to the equations of ANZELIUS and NUSSELT, when their particular boundary values of $\theta_o(\eta)$ and $\tau_o(\alpha)$ are substituted.

Uniform Supply Inlet Temperature and Uniform Initial Wall Temperature.

Particular solutions of eqn. (4.7) are obtained if the boundary conditions of eqns. (4.5) and (4.6) become,

$$\theta = \text{constant} \quad \text{at} \quad \xi = 0$$

$$\text{and } \tau = \text{constant} \quad \text{at} \quad z = 0 \, (\eta = 0).$$

Therefore, if for a gas blow, these boundary conditions are

$$\theta = 1 \quad \text{at} \quad \xi = 0,$$

$$\text{and} \quad \tau = 0 \quad \text{at} \quad z = 0, \, (\eta = 0).$$

eqn. (4.7), is now

$$\theta(\Lambda, \eta) = e^{-\Lambda-\eta}\left[e^{\eta} + \sqrt{\Lambda}\int_0^{\eta} \frac{e^{\alpha}}{\sqrt{\eta-\alpha}} \cdot I_1\left\{2\sqrt{\Lambda(\eta-\alpha)}\right\} d\alpha \right] . \qquad (4.8)$$

For an air blow however, the boundary conditions could be written,

$$\theta_1 = 0 \quad \text{at} \quad \xi = 0$$

$$\text{and} \quad \tau_1 = 1 \quad \text{at} \quad z = 0, \, (\eta = 0),$$

and eqn. (4.7) now gives

$$\theta_1(\Lambda, \eta) = e^{-\Lambda-\eta}\int_0^{\Lambda} e^{\alpha} I_o\left\{2\sqrt{\eta(\Lambda-\alpha)}\right\} d\alpha \quad . \qquad (4.9)$$

It has been shown by H.M. Melvin, that for the same values of Λ, and η,

$$\theta(\Lambda, \eta) = 1 - \theta_1(\Lambda, \eta) \quad , \qquad (4.10)$$

which is a well-known result for these simpler boundary conditions. The practical significance is that for these boundary conditions one set of theoretical curves, e.g. those for a gas blow, can be transformed into those for an air blow by applying the relationship of eqn. (4.10). The curves of Fig. 11 have been calculated for such boundary conditions,

viz. $\theta = 1$ at $\xi = 0$

and, $\tau = 0$ at $\eta = 0$.

Use of Analysis at $\eta = 0$.

Eqn. (4.7) can be simplified in order to act as a check on the extrapolation used in the determination of the experimental heat transfer coefficients.

When $\eta = 0$, the gas outlet temperature from the test section, i.e. $\xi = \Lambda$,

$$\theta(\Lambda, 0) = e^{-\Lambda}\left[\theta_0(0) + \int_0^{\Lambda} \tau_0(\alpha)\, e^{\alpha}\, d\alpha\right] \quad . \qquad (4.11)$$

Integration by parts, gives

$$\int_0^{\Lambda} \tau_0(\alpha)\, e^{\alpha}\, d\alpha = e^{\alpha}\, \tau_0(\alpha)\Big|_0^{\Lambda} - \int_0^{\Lambda} e^{\alpha} \cdot \frac{d}{d\alpha}\left[\tau_0(\alpha)\right] d\alpha \quad . \qquad (4.12)$$

This expression can be evaluated, if it is assumed that at $\eta = 0$, the wall temperature gradient is linear,

i.e. $\dfrac{d}{d\alpha}\left[\tau_0(\alpha)\right] = K$,

where K may have any value, positive or negative. K is determined from the inlet and outlet wall temperatures.

i.e. $$K = \frac{\tau_0(\Lambda) - \tau_0(0)}{\Lambda} \quad . \qquad (4.13)$$

The resulting expression for the integral of eqn. (4.12) is incorporated in eqn. (4.11), to give ,

$$\theta(\Lambda, 0) = e^{-\Lambda}\left[\theta_0(0) - \tau_0(0) + K\right] + \tau_0(\Lambda) - K \quad . \tag{4.14}$$

All these temperatures are values at $\eta = 0$; $\theta_0(0)$ and $\theta(\Lambda, 0)$ are the gas inlet and outlet temperatures respectively, while $\tau_0(0)$ and $\tau_0(\Lambda)$ are the wall temperatures at inlet and outlet respectively.

Eqn. (4.14) is rearranged to give an expression for the non-dimensional length Λ , in terms of these inlet and outlet temperatures, viz.,

$$\Lambda = \log_e \left\{ \frac{\theta_0(0) - \tau_0(0) + K}{\theta(\Lambda, 0) - \tau_0(\Lambda) + K} \right\} \quad . \tag{4.15}$$

CHAPTER V

SINGLE BLOW TEST RESULTS

This investigation is confined to heat transfer to and from gases whose flow conditions are laminar, i.e. at some position within the test length fully-developed laminar flow is established. Therefore, in this consideration of the values of heat transfer coefficients obtained in these tests, it is first shown that this condition of laminar flow is satisfied.

From energy considerations within the passage length, and at entry to it, a theoretical expression is obtained for the pressure loss due to entry conditions and the establishment of laminar flow within the test length. This expression, derived initially for steady state conditions with no heat transfer, is developed to include also unsteady state conditions with moderate temperature differences. The overall theoretical pressure loss for the test section will be this entry loss plus the loss due to the viscosity of the gas.

Actual pressure drop values have been determined during steady and unsteady state tests.

Pressure drop coefficients calculated from the theoretical and the actual pressure losses are plotted against Reynolds number. A comparison between the curves obtained indicates that, in the tests quoted, the gas flow is laminar. Agreement between the experimental results for the steady and the unsteady state justify the use of these expressions in the unsteady state results.

The method used for determining the heat transfer coefficient is not original andreference is made in the second section of this chapter to those responsible for its development. A more detailed description is presented in Appendix IV, along with a specimen calculation.

From the experimental temperature records, boundary conditions for substitution in the theoretical analysis are obtained. It is then possible to calculate the theoretical gas outlet temperature for a range of values of non-dimensional length and time. A comparison of the experimental gas outlet temperature record and these calculated curves determines the heat transfer coefficient. By the assumptions of the analysis, this heat transfer coefficient is based on local mean temperature difference, is constant over the test length, and has been constant from the time of origin of the unsteady state process.

Values of heat transfer coefficients, obtained in this way are presented in graphical form for both "gas" and "air" single blow tests.

A form of arithmetic mean temperature difference has also been introduced and this chapter contains an appreciation of the test transfer coefficients, based on this temperature difference, determined from the single blow tests.

1. PRESSURE DROP ACROSS TEST SECTION

Pressure Drop in Steady Flow through a Smooth Tube.

For the simple case of a plain tube through which a gas flows steadily, and no heat exchange takes place between gas and wall, the general expression for pressure drop is,

$$\Delta p = \phi \cdot \frac{L}{d} \cdot \rho \frac{v_M^2}{2g} \quad . \tag{5.1}$$

For laminar flow in smooth tubes, the pressure drop coefficient, $\phi = \frac{64}{Re}$.

In addition, there is a loss at entry due to the change in cross-section and the development of laminar flow. The value of this loss, established in Appendix III, in eqn. (A3.3), is, $\Delta p_{1-3} = \rho \frac{v_{M3}^2}{2g}\left(K_{eb_3} - K_{eb_1}\sigma^2\right)$

$$\therefore \Delta p_{total} = \phi' \cdot \frac{L}{d} \cdot \rho \frac{v_M^2}{2g} \tag{5.2}$$

where $$\phi' = \phi + \left(K_{eb_3} - K_{eb_1}\sigma^2\right) d/L \quad . \tag{5.3}$$

The kinetic energy velocity distribution coefficients K_{eb_1} and K_{eb_3} are evaluated in Appendix III. ϕ' is therefore the theoretical value, which includes the loss at entry to the passage, and the transformation from a square to a round cross-section,

$$\therefore \quad \phi' = \frac{64}{Re} + \left(2.00 - 1.60\,\sigma^2\right) d/L \quad .$$

For the apparatus used, the contraction area ratio, $\sigma = 0.3913$, i.e. $\sigma^2 = 0.1532$ and $d/L = 1/150$,

$$\therefore \phi' = \frac{64}{Re} + 0.0117 \quad . \tag{5.4}$$

This value is used in the expression for the overall theoretical pressure drop in the tube, including loss at entry,

$$\Delta p_{total} = \phi' \cdot \frac{L}{d} \rho \frac{v_M^2}{2g} \quad . \qquad (5.2)$$

<u>Pressure Drop Readings as an Indication of Laminar Flow Conditions.</u>

<u>Isothermal</u> steady flow tests were made under several conditions, (i) with single connection from "gas" blower to heat transfer section, and (ii) with selector valve box fitted, (a) supply from "gas" blower

(b) supply from "air" blower.

In all cases the air was cold and readings were taken for varying mass flows.

The coefficient ϕ_{exp} was determined by means of equation (5.1) and Fig. 15 shows the curves obtained by plotting these values against R_e . The discrepancy between the results for the single connection and those for the valve box are presumably due to the position of the tapping point, and possibly to the beneficial effect of a slight leakage past the edges of the "closed" valve plate.

Whereas the theoretical correction for entry losses was shown by eqn. (5.4) to be 0.0117, the isothermal experimental results of Fig. 6 give 0.0140 as an average value. The discrepancy in these values can be traced partly to the slight difference in the gas and air supply conditions, as indicated by the spreading of these corresponding points with increase of R_e in Fig. 15.

A further test, in which the mass flow through the test section was estimated from the magnitude of the mass flow at the supply orifice, shows in Fig. 16 the sudden increase of C, a coefficient proportional to ϕ_{exp}, at a certain definite flow rate. The Reynolds number corresponding to this rate was estimated to be 3100 - 3200.

The consistency of these results indicates that fully developed laminar flow does exist in the arrangement beyond the entry length at values of $R_e > 2500$, and that the onset of turbulence could be clearly indicated by measurements of the pressure drop. This was confirmed later in heating tests at $R_e > 2750$, when the record of the outlet temperature from the test section became very irregular in form.

In testing therefore, great importance was attached to the pressure drop readings as an indication of laminar flow.

Pressure Drop in Unsteady State Conditions.

For unsteady state conditions, and also steady state non-isothermal conditions, an estimate of the theoretical overall pressure drop coefficient ϕ' can also be made.

When a hot gas passes through a cooler passage, the gas is itself cooled and therefore its velocity decreases as the gas travels through this passage. Within the test length the velocity at any cross-section can be considered to depend on the local mean gas temperature.

Therefore the pressure drop between inlet and outlet of test section, in addition to that resulting from the viscosity of the gas, derived as in Appendix III, becomes,

$$\Delta p_{1-4} = \frac{\rho}{2g}\left(K_{eb_4} v_{M_4}^{\ 2} - K_{eb_1} v_{M_1}^{\ 2}\right)$$

$$= \frac{\rho v_{M_3}^{\ 2}}{2g}\left(K_{eb_4} \frac{v_{M_4}^{\ 2}}{v_{M_3}^{\ 2}} - K_{eb_1} \frac{v_{M_1}^{\ 2}}{v_{M_3}^{\ 2}}\right) \qquad (5.5.)$$

where point 1 is at the square cross-section preceding inlet, and point 4 is at outlet from the test section.

The velocity ratios of eqn. (5.5) are now replaced by the ratios of the mean gas temperatures at the same positions,

i.e. $\dfrac{v_{M_4}}{v_{M_3}} = \dfrac{\theta_4}{\theta_3}$ and $\dfrac{v_{M_1}}{v_{M_3}} = \dfrac{v_{M_1}}{v_{M_2}} \cdot \dfrac{v_{M_2}}{v_{M_3}} = \sigma \dfrac{\theta_2}{\theta_3}$.

Note. The inlet temperature measured in the unsteady state tests is at point 2, at the mouth of the circular tube, and not at point 1 which is a short distance upstream.

Eqn. (5.5) can now be written,

$$\Delta p_{1-4} = \rho \frac{v_{M_3}^{\ 2}}{2g}\left[K_{eb_4}\left(\frac{\theta_4}{\theta_3}\right)^2 - K_{eb_1} \sigma^2 \left(\frac{\theta_2}{\theta_3}\right)^2\right] . \qquad (5.6)$$

Point 3 is a position along the tube, at a cross-section where the parabolic velocity distribution is established. Point 4 is at the end of this tube, i.e.

further downstream, and has the same cross-section, and therefore for moderate temperature differences,

$$K_{eb_4} = K_{eb_3} = 2.0$$

$$K_{eb_1}\sigma^2 = 1.6 \times 0.1532 = 0.245$$, as in eqn. (5.4).

Therefore in the unsteady state case the value of ϕ' to be used in eqn. (5.2.) is,

$$\phi' = \frac{64}{Re} + \left[2 \times \left(\frac{\theta_4}{\theta_3}\right)^2 - 0.245\left(\frac{\theta_2}{\theta_3}\right)^2\right]\frac{d}{L} \quad . \qquad (5.7)$$

Pressure Drop Coefficients - Single Blow, Heating and Cooling.

Pressure drop coefficients were determined from readings taken during the heat transfer tests. After being corrected for the density effect mentioned earlier, these values are corrected to approximately isothermal conditions using the formulae derived previously.

Theoretically, $\phi'_{non\ isothermal} = \frac{64}{Re} + \left[2\left(\frac{\theta_4}{\theta_3}\right)^2 - 0.245\left(\frac{\theta_2}{\theta_3}\right)^2\right]\frac{d}{L}$ (5.7)

and, $\phi'_{isothermal} = \frac{64}{Re} + 0.0117$. (5.4)

$\therefore \phi_{exp}$ corrected to isothermal conditions is given by the following expression, $\phi_{exp} + 0.0117 - \left[2\left(\frac{\theta_4}{\theta_3}\right)^2 - 0.245\left(\frac{\theta_2}{\theta_3}\right)^2\right]\frac{d}{L}$.

These values are plotted in Fig. 17 and compared with the curve for ϕ_{exp} obtained in Fig. 15 for isothermal conditions, i.e., ϕ_{th} .

The agreement between these values is considered as proof that laminar flow was established within the test section, during these experiments.

2. HEAT TRANSFER COEFFICIENTS

Calculation of Heat Transfer Coefficient.

In principle the method of determining the heat transfer coefficient from any set of experimental results is to compare the corrected outlet temperature with a corresponding set of calculated curves.

This method which originated with SCHUMANN, has been developed by SAUNDERS & FORD, and also by JOHNSON. In Appendix IV the procedure adopted is described in more detail, and a specimen calculation is made.

Influence of Varying Supply Inlet Temperature and Tube Initial Temperature Distribution.

When the inlet temperature does vary as, say, in Fig. 6 for Test F2, a first assumption of uniform gas inlet temperature might be made, this being probably sufficiently accurate for this particular test. A comparison of the Nusselt numbers obtained in this way, and also from the exact inlet temperature variation, showed that the simpler method overestimated the Nu value, during the early part of the blow.

In all the heating tests, the relation between reduced gas inlet temperature and reduced time η, has been found to be approximately the same, and from this boundary condition and the analytical expressions of Appendix II a series of curves of reduced temperature against reduced length Λ was plotted with η/Λ as parameter.

The reduced temp. $\Theta = \dfrac{\theta - \tau_M}{\theta_i - \tau_M}$

where τ_M = tube initial mean temperature

θ_i = time average supply inlet temperature.

These curves are shown in Fig. 12, and may be compared with those of Fig. 11, in which the assumption of uniform inlet temperature has been adopted.

In the cooling tests, two fairly general curves of reduced inlet temperature against reduced time, applied.

It is also necessary to consider the temperature distribution along the tube at the instant of changeover. In the heating tests the temperature can be assumed uniform along the tube and this assumption was made for the calculations for Figs. 11 and 12.

In the single air blow and regenerator cyclic tests, this assumption is no longer valid, and it was for this reason that the tube temperature, $\tau_{\eta=0} = \tau_0(\xi)$, and not $\tau_{\eta=0}$ = constant, was used in the theoretical analysis. In these single blow tests, two initial temperature distributions, assumed linear, were found necessary from the temperature records and were used in the calculations, viz.,

$$\tau_0(\xi) = 1{\cdot}31 - \frac{0{\cdot}62}{\Lambda}\cdot\xi \qquad (5.8)$$

and

$$\tau_0(\xi) = 1{\cdot}2 - \frac{0{\cdot}4}{\Lambda}\,\xi \quad . \qquad (5.9)$$

New curves were calculated with these boundary conditions, and one set, assuming such an initial tube temperature distribution and uniform air inlet temperature, is shown in Fig. 13.

Values of Heat Transfer Coefficient obtained in Single Blow Tests.

The first striking factor of the values of heat transfer coefficient obtained is that, in both air and gas blows, they increase with time (Fig. 18a). In the gas blow generally they tend to a maximum within the time of the test. Any difference of form in the two cases is most likely due to the differing effect of heat losses and the variation in tube temperature distribution discussed later. The possibility that part of this increase of h with time may be apparent only, and may, in fact, be due to the effect of temperature gradient in the tube wall normal to the direction of flow, which has been neglected in this analysis, cannot be discounted completely.

A non-dimensional representation of the form of h is considered briefly in Appendix V.

Nusselt Numbers obtained in Single Blow Tests, based on Local Temperature Difference.

The mean temperature on which air property values have been based in these experiments is the average of the time means of both inlet and outlet temperatures for the eight minute period under consideration. The actual property values are taken from the tables of KEENAN & KAYE. For each test a mean Reynolds number and curve of Nusselt number with time can be determined.

By extrapolation, the Nusselt numbers for the gas blow tests at $\eta = 0$ have been found and these are plotted in Fig. 19. These particular values have been obtained assuming uniform initial tube temperature and uniform gas inlet temperature. The fact that these values are not uniform particularly at lower R_e has been taken into account in Fig. 20, which shows a much improved correlation.

The values for the air blow however at $\eta = 0$, show a marked difference in Fig. 19, these being much lower at low R_e.

Examination of the typical temperature records of Figs. 7 and 8 does give a possible explanation. Here, it is seen that for a period immediately following change-over to cold supply, at exit, the temperature profile is the reverse of what might be expected at first sight. At this time the temperature distribution along the tube wall is of the form shown in Fig. 21, so that whilst being heated in the first sections of the tube, the air is giving up heat in the later sections. In fact, the effective heating length of the tube has been reduced because of its temperature distribution. Except at the very lowest flow rates it is found that when the tube temperature is approximately uniform, the temperature profile reverts to its accepted shape for a fluid cooling a circular tube wall.

If then, this instant of uniform tube temperature is taken as the origin of time measurement and the Nu_l values for $\eta = 0$ are recalculated, the values obtained are shown in Fig. 22, along with the curve for gas blow values of Fig. 20. The variation of h_l with η is now as shown in Fig. 18b.

These Nusselt values are of course, local values based on the assumption that they are constant along the length. On the other hand, theoretical values of Nu_l for steady state conditions and uniform wall temperature were shown in Chapter II to vary along the length from a value of ∞ at $x = 0$, to $Nu = 3{\cdot}65$ at $x = \infty$, and to be given by

$$Nu = f\left(\frac{R_e P_r}{L/d}\right) .$$

Therefore the overall values for steady state, equivalent to the experimental values of Fig. 20 can be expected to be greater than 3.65 and to increase with R_e for a constant length. This does conform with the results of Fig. 20.

The general form of Nu_l with time varies only slightly for heating and cooling and the values at $\eta/\Lambda = 0{\cdot}3$ are shown in Fig. 23. The very low values for cold supply correspond to those tests in which, even after this time, the temperature profile is still reversed. This phenomenon is indicated in Fig. 8.

From the variation of h_l with time in Fig. 18, it is assumed that for other η values, i.e. $0 < \eta < 0{\cdot}3\Lambda$, Nu_l is given for both air and gas blow by an expression

$$Nu_l = \left(1 + \frac{2}{3} \cdot \frac{\eta}{\Lambda}\right) Nu_{li} \qquad (5.10)$$

where Nu_{li} is the value of Nu_l at $\eta = 0$, when the tube temperature is uniform, obtained from the curve of Fig. 20 or 22.

At $\eta/\Lambda = 0.3$, the tube has now adopted a more or less uniform temperature gradient (except for air blow with low R_e) and the experimental Nu_l values can be compared with the theoretical $Nu_l = \frac{48}{11}$, for steady state. To the latter value, of course, the resulting increase of Nu_l with entry length must be added. The effect of the preceding period in the unsteady state case must be considered when comparing such results with those for steady state operation.

<u>Nusselt Numbers based on Arithmetic Mean Temperature Difference.</u>

From the heat transfer coefficient h_{am}, determined as shown in Appendix VI, corresponding values of Nu_{am} are obtained.

These also vary with time and, as with the Nu_l values, a difference exists between values for air and gas blows, as is shown in Fig. 24 for the values of Nu_{am} at $\eta = 0$.

If as before, the origin of time is taken as that instant when the tube temperature is approximately uniform, there is excellent agreement between the results for both cases, which are incorporated in Fig. 1.

At this stage, with $\eta = 0$, the effects of convection will not be evident. However, even at $\eta > 0$, the

Grashof numbers are not very great (say 1.4×10^4 for gas blow, and 5×10^3 for air blow) and the effect of convection is unlikely to be any greater than the error introduced by assuming uniform tube temperature.

The theoretical lines in Fig. 1 are for different values of the parameter $\left(\frac{G_r P_r d}{L}\right)_s$, viz., 0 and 10^4, and the limit line $Nu_{am} = \frac{2}{\pi} G_z$. Corresponding values of the above parameter for the gas and air blows are estimated to be 65 and 25 respectively.

From these results it is possible to rule out any marked influence of convection in these experiments.

At $\eta/\Lambda > 0$ the Nu_{am} values will diverge from the above curve in a manner which is, most likely, due to the modified temperature distribution in the tube, and as suggested in Appendix V, it may be that some relationship such as

$$Nu_{am} = f\left(R_e, P_r, \eta/\Lambda\right) \tag{5.11}$$

applies here also. It is felt however, that the value of this coefficient at the moment, is restricted to the conditions of uniform tube temperature and $\eta = 0$, and under these conditions the results obtained justify the experimental arrangement and the method adopted.

<u>Overall Accuracy of Data</u>.

It is practically impossible to estimate the overall accuracy because of this correction for heat losses. A general balancing of the methods adopted suggests that the maximum error in the results obtained is unlikely to exceed ± 7 or 8%.

CHAPTER VI

PARALLEL FLOW REGENERATOR

For practical reasons, the circular cross-section passage used in this investigation is unlikely to be used in an actual regenerative heat exchanger. The regenerator experiments were made with a view to comparing the heat transfer processes in single blow and cyclic operation.

The heat transfer coefficients are in this case obtained from a comparison of values of regenerator efficiency calculated theoretically and values derived from the experimental results.

The regenerator experiments were not carried out over a range of Reynolds number but over a range of period times. Some agreement is obtained between the single blow and regenerator results.

1. THEORETICAL ANALYSIS OF REGENERATOR EFFICIENCY.

Regenerator Efficiency for Parallel Flow Operation.

As defined by HAUSEN, the regenerator efficiency, E_{reg}, is the ratio between the heat quantity transferred from one fluid to the other, and the heat which would be necessary to raise the inlet temperature θ_2 of the cold supply up to the inlet temperature θ_1 of the hot supply.

By assuming that,

(i) the theoretical conductivity of the tube material is infinitely great in the direction normal to the gas flow and negligible in the direction parallel to the flow,

(ii) the values of η and of Λ in both periods are equal, the heat transfer coefficients being constant along the length,

(iii) the gases enter the regenerator with a uniform and constant temperature,

(iv) any gas introduced during one period passes through the regenerator within the duration of that period,

HAUSEN showed that, for parallel flow operation,

$$E_{reg} = \frac{1}{2}\left(1 - \frac{\theta_{M\Lambda}}{\theta_2}\right) \qquad (6.1)$$

where

$$\frac{\theta_{M\Lambda}}{\theta_2} = \sum_{n=1,3,5,\ldots}^{\infty} \frac{8}{(n\pi)^2} \cdot e^{\frac{-n^2}{\psi^2+n^2}\Lambda} \cdot \cos\left(\frac{\psi n}{\psi^2+n^2}\Lambda\right) \qquad (6.2)$$

and $\psi = \frac{1}{\pi} \times$ the non-dimensional period time η.

It would also be possible to define regenerator efficiency in terms of the heat given to the hot supply.

In terms of heat quantities,

$$E_{reg} = \frac{Q}{Q_{id}} \qquad (6.3)$$

where Q = heat to or from supply flow,

Q_{id} = ideal heat quantity available, which is the product of the water equivalent of the gas flowing in one period and the difference between the two supply inlet temperatures.

Regenerator Efficiency with Infinitely Short Blow Times.

The regenerator efficiency for parallel flow operation and infinitely short blow times, can be evaluated simply from eqn. (4.7), if the non-dimensional length Λ is the same for each period.

i.e. $\eta_H = \eta_c = 0$

$\Lambda_H = \Lambda_c$

Gas inlet temp., $\theta_{o_H} = 1.$

Air inlet temp., $\theta_{o_c} = 0.$

Subscript H refers to hot supply or gas conditions, and subscript c to cold supply or air conditions.

Under these conditions, it is seen by inspection, that the tube temperature will be constant in both periods and uniform throughout the test length, and of magnitude,

$$\tau_H = \tau_c = 1/2 \quad .$$

Therefore, from eqn. (4.7) the fluid outlet temperatures are

$$\theta_H(\Lambda) = e^{-\Lambda}\left(1 + \tfrac{1}{2}e^{\Lambda} - \tfrac{1}{2}\right) = \tfrac{1}{2}\left(e^{-\Lambda} + 1\right) \qquad (6.4)$$

and $\theta_c(\Lambda) = \frac{1}{2}\left(1 - e^{-\Lambda}\right)$, similarly. (6.5)

The heat transferred to the air in one period is,

$$Q = W_c z_c \left[\theta_c(\Lambda) - \theta_{oc}\right] , \qquad (6.6)$$

and therefore from eqn. (6.5)

$$Q = W_c z_c \cdot \tfrac{1}{2}\left(1 - e^{-\Lambda}\right) \quad .$$

The ideal quantity of heat available for transfer is,

$$Q_{id} = W_c z_c \left[\theta_{OH} - \theta_{OC} \right] , \qquad (6.7)$$

$\therefore \; Q_{id} = W_c z_c$, in this case.

Substitution of these values for Q and Q_{id} in eqn. (6.3), gives an expression for the efficiency of a parallel flow regenerator with infinitely short blow times,

$$E_{reg} = \frac{1}{2}\left(1 - e^{-\Lambda}\right) \quad . \qquad (6.8)$$

This expression is also obtained from eqns. (6.1) and (6.2) by substituting the condition $\eta = 0 \; , \therefore \psi = 0$.

$$\therefore \; E_{reg} = \frac{1}{2}\left[1 - \sum_{n=1,3,5,\dots}^{\infty} \frac{8}{(n\pi)^2} \cdot e^{-\Lambda} \right]$$

$$= \frac{1}{2}\left[1 - \frac{8}{\pi^2} e^{-\Lambda} \cdot \sum_{n=1,3,5\dots}^{\infty} \frac{1}{n^2} \right] \quad . \qquad (6.9)$$

The series within this expression can be written,

$$\sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} = \sum_{n=1,2,3,\dots}^{\infty} \frac{1}{(2n-1)^2} \quad ,$$

and evaluated using the series for $\cos x$,

$$\cos x = \prod_{n=1}^{\infty} \left\{ 1 - \frac{4x^2}{(2n-1)^2 \pi^2} \right\} = 1 - \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} x^2 + \dots \quad .$$

Comparison of this series with the other series for $\cos x$, gives

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8} \quad .$$

Eqn. (6.9) now becomes

$$E_{reg} = \frac{1}{2}\left(1 - e^{-\Lambda}\right) \quad ,$$

which is the same as eqn. (6.8).

2. REGENERATOR TESTS AND RESULTS

Cyclic Operation as a Parallel Flow Regenerator.

Preliminary preparations are similar to those for single blow operation, although particular care is necessary in order that supply from either blower is not affected by changeover of the valves.

Initially it was arranged that the valves should be operated automatically on a fixed time sequence; this was later discarded in favour of the simplicity of hand operation.

When the hot supply, exhausting to atmosphere has reached a steady temperature, the valves are changed over and the sequence initiated. The gas and air periods can be made of any duration required.

After continuous cyclic operation for some time, the condition is reached in which each successive gas period produces a similar temperature record. From readings taken during subsequent consecutive air and gas periods, heat transfer coefficients are calculated.

The temperature record from such a test is shown in Fig. 25.

Heat Loss in Cyclic Operation.

Cyclic operation was taken as the basis for fixing the flow rate through the annular space for single blow tests in the following way. The heat loss during any period was determined as for single blow tests. It was considered then that if, after continuous operation, the heat loss, during two consecutive air and gas periods of the same duration, was the same, then the flow conditions in the annulus and inner tube were similar.

While the actual time for two such periods may be the same, the corrected times obtained by the heat loss correction method will be quite different. Provided the heat loss is not too high a percentage of the heat being exchanged, it would be possible to alter the period times during any experiment so that the two corrected times became approximately the same. Where the heat loss is comparatively great as in this apparatus, this was considered to be an over-elaboration. Whilst it is possible to correct for considerable heat losses in single blow operation, the effect of heat losses in cyclic operation can be corrected only in a superficial manner.

The heat given to the cold supply is proportional to the area enclosed on the temperature record by the inlet and outlet temperatures during the air blow. By definition, the ideal heat quantity available is the water equivalent of the supply per period multiplied by the difference between the inlet temperatures of both periods. There is however, a heat loss, over the two periods, equal to the difference between the heat given up by the hot supply and the heat received by the cold.

It is considered then that in the experiment, <u>without</u>

heat loss, for E_{reg} based on air blow, Q_{id} is the water equivalent of the cold supply multiplied by the area between the inlet temperatures of both periods, minus the heat loss in both periods, as shown in Fig. 25. In the corresponding expression for E_{reg} based on gas blow, the heat loss in both periods is added and not subtracted.

Values of Nu_l from Regenerator Experiments.

A comparison of the values of E_{reg} calculated from experimental results using eqn. (6.3), and with the theoretical results of eqns. (6.1) and (6.2), gives a value of a mean Nusselt number. This is a value which is assumed constant along the length of the tube and for the extent of the period, but is calculated on the basis of local temperature difference. This is analogous to the Nu_l value of the single blow tests, but it cannot be expected to compare directly because of the reversed temperature profile of the outlet air during the cold supply period.

The mean values are tabulated below, and compared with a mean value, predicted from the single blow tests results for approximately similar conditions. All these tests were carried out at approximately the same R_e values, 1800 for gas and 1900 for air, with equal experimental period times. A tendency of the mean Nu_l value to increase with increase of η/Λ is just apparent.

Test	Actual Period Mins.	Gas Blow			Air Blow			Mean η/Λ	Mean Nu_l Regen?	Mean Nu_l Single Blow
		η/Λ	Nu_l	Re	η/Λ	Nu_l	Re			
U_2	2	0.14	4.96	1760	0.20	4.36	1900	0.17	4.66	4.8
U_3	3	0.22	4.97	1780	0.32	4.71	1870	0.27	4.84	5.3
U_1	4	0.31	5.14	1830	0.51	4.6	1930	0.41	4.87	5.6
U_4	5	0.41	5.00	1800	0.62	4.84	1910	0.51	4.92	5.8
U_5	6	0.48	5.1	1820	0.72	4.87	1920	0.60	4.98	6.0

It would be possible to consider each individual blow period of cyclic operation, as a single blow period and analyse it accordingly. This was not done, partly because examination of the regenerator results showed no obvious irregularities, and partly because it was felt that the results did not merit this elaboration.

Values of Nu_{am} from Regenerator Experiments.

The cyclic results were, however, reduced to single blow periods, for the calculation of heat transfer coefficients based on arithmetic mean temperature difference. This gives separate Nu_{am} values for air and gas blow and their mean was found to agree fairly well with that estimated from the single blow test results, for similar temperature conditions.

Test	Actual Period Mins.	Gas Blow		Air Blow		Mean Nu_{am} Regenerator.	Mean Nu_{am} Single Blow.
		Nu_{am}	R_e	Nu_{am}	R_e		
U_2	2	4.33	1760	2.84	1900	3.58	3.45
U_3	3	3.91	1780	3.21	1870	3.56	3.7
U_1	4	4.07	1830	3.20	1930	3.64	4.0
U_4	5	4.05	1800	3.36	1910	3.71	4.2
U_5	6	3.99	1820	3.35	1920	3.65	4.3

In a method which is attributed to HEILIGENSTAEDT, this type of arithmetic mean temperature difference is used to predict the total heat exchanged in a regenerator.

i.e. $$Q = \frac{1}{\frac{1}{h_H Z_H} + \frac{1}{h_c Z_c}} \cdot PL\left(\theta_H - \theta_c\right) \qquad (6.10)$$

where, θ_H = mean of average inlet and outlet gas temperatures over whole period.

h_H = heat transfer coefficient for gas based on arithmetic mean temperature difference.

Z_H = duration of gas blow.

suffix c refers to cold supply or air conditions.

This expression has a limited application in design work.

CHAPTER VII

PREVIOUS INVESTIGATIONS.

Previous investigations of unsteady state heat transfer in laminar flow, have been principally confined to the conditions which exist in regenerators of the contra-flow type. The general pattern of such investigations has been similar, in that some theoretical method of prediction of outlet temperature has been developed, and this in turn applied to experimental results to obtain values of Nusselt number. A similar pattern exists in this investigation except that some of the boundary condition restrictions are avoided.

1. THEORETICAL WORK

Temperature Variation.

Analytical solutions of the differential eqns. (A2.1a) and (A2.2a) obtained in Appendix II, for heat transfer between the gas and tube surface, and also between the tube surface and the tube itself,

i.e. $$\theta - \tau = -\frac{\partial \theta}{\partial \xi}$$

and $$\theta - \tau = \frac{\partial \tau}{\partial \eta}$$

have been obtained by ANZELIUS, NUSSELT, HAUSEN, SCHMEIDLER, and others for different sets of boundary conditions.

Anzelius and Nusselt dealt with the single blow period, whilst Nusselt, Hausen, and Schmeidler and others produced results applicable to the cyclic operation of a regenerator. A fairly comprehensive bibliography of this work is given by ILIFFE.

Of the regenerator analyses above, only Hausen has considered the case of parallel flow, and it is his theory which was used earlier in the calculation of Nu_l values for cyclic operation.

With the advent of the practical gas turbine, a new interest arose in the regenerator and additional analyses of the cyclic case have been produced.

Probably the most convenient of all the methods, suitable for cyclic or single blow operation, is that of SAUNDERS & SMOLENIEC, and ALLEN. This is a partly iterative method in which the wall temperatures are given by finite difference approximations of the governing equations. With modifications to the actual method presented, any temperature boundary conditions can be applied.

The analytical method developed in Appendix II was preferred for this investigation, because it was felt that some applications might exist in which the particular boundary conditions would themselves suggest a possible approximation. The evaluation of gas outlet temperature for specific boundary conditions and specific values of η and Λ is easier if the analytical method is used.

For regenerator calculations, however, it is unlikely that unless the result is already available in tabulated form, there is any more convenient method than that of SAUNDERS & SMOLENIEC.

A similar type of method was used by JOHNSON for single blow calculation; his experimental work is referred to later.

Heat Transfer Coefficient.

There is no theoretical work regarding the heat transfer coefficients which may be expected under unsteady state conditions. This is a result of the complex variation of the boundary conditions.

The energy equation for the fluid flow would require to be solved, with the conditions,

$$\theta = \tau_o(\xi) \quad \text{for} \quad \eta = 0 \; , \quad r = s \quad \text{and} \quad \xi > 0 \; ,$$
$$\theta = f[\eta, \xi, \tau_o(\xi)] \quad \text{for} \quad \eta > 0 \; , \quad r = s \quad \text{and} \quad \xi > 0 \; ,$$
$$\text{and} \quad \theta = \theta_1 \quad \text{for} \quad \eta \geq 0 \; , \quad r = s \quad \text{and} \quad \xi = 0 .$$

Provided the temperature difference, and therefore the acceleration and heat storage terms, are not too great, the theoretical results for Nusselt numbers in steady state flow should apply in unsteady state, if the boundary conditions are the same.

This means that in the case of a circular passage in a contra-flow regenerator, where at all times after the initial heating, the temperatures along the wall will be approximately linear, but not uniform, the theoretical Nusselt number should be $48/11$ and independant of R_e . This was assumed by COX.

2. HEAT TRANSFER EXPERIMENTS.

The particular problem of determining heat transfer coefficients in unsteady state heat transfer in regenerators was one of the items of the GENERAL DISCUSSION ON HEAT TRANSFER, 1951. Here Professor Saunders expressed the following preferred order of attack,

steady state measurement,

cyclic operation,

single blow method.

The steady state method is possible in a single flow passage and the results for a circular cross-section are well known. For the type of matrix, most likely to be used in future regenerators, this method is not practicable.

Regenerator packings may be of several types - loose solids, wire gauze, expanded metal and flame trap material. Any packing which results in irregular flow passages is outwith the compass of this discussion.

Single Blow Tests by JOHNSON.

JOHNSON experimented with flame trap material of triangular cross-section passages. He employed the single blow method, and the method adopted here, for obtaining values of heat transfer coefficients from the theoretical calculations, follows his method fairly closely.

His tests were made with a matrix, uniformly cold initially, through which hot gas was passed. His experiments were within the range, $10 < R_e < 200$.

For this flame trap material he noted a tendency for the values to become constant at $Nu_l = 2$, at $R_e > 100$.

With this particular uniform initial matrix temperature, it cannot be expected that Nu will reach the theoretical value assuming uniform temperature gradient along the passage length, viz. $Nu_l = 3.07$. ($48/11$ is the equivalent value for a circular passage).

Cyclic Operation Tests by COX & LAMB.

In these tests a similar type of flame trap material was used in a contra-flow regenerator.

By determining first the regenerator efficiency from experiment, and comparing this with theoretical values, which had been corrected for the duration of blow period and the thermal conductivity of metal parallel to the direction of gas flow, Cox determined values of Nu_l .

From his results, Cox concluded that the Nusselt number, which is a combined value for heating and cooling, varies with Reynolds number, according to the relationship,

$$Nu_l = 2.4 + 0.12 \frac{R_e}{L/d} \quad . \qquad (7.1)$$

This increase of Nu_l with R_e is, he submits, a result of the entry length in the passages. The value of 2.4 is about

20% less than his accepted theoretical value for laminar flow in a triangular cross-section passage with uniform wall temperature gradient, i.e. $Nu_l = 3.07$.

These experiments were carried out in the range, $40 < R_e < 400$, for two different values of the ratio h/Λ , i.e. approximately 0.2 and 0.4. No significant difference between the results obtained for tests with these different ratios seems to have appeared.

Experiments by GLASER, and by LUND and DODGE.

Other experiments on regenerators using regular passages have been carried out by GLASER and also by LUND & DODGE. In both of these sets of experiments, Frankl packing was employed in practical contra-flow regenerators of the type used in gas liquefaction processes. This particular packing is composed of diagonal corrugated strip without any intermediate flat strip, as in flame trap material.

Glaser adopted cyclic operation but determined heat transfer coefficients based on the arithmetic mean temperature difference as indicated in eqn. (6.10).

In the Lund and Dodge experiments, single blow operation was used, results being taken from the gas blow only. Here too, Nu_{am} values were obtained.

Comments.

As the initial matrix temperature distribution differs in the experiments of JOHNSON and of COX, their results cannot be compared. It must be noted that in both these cases, experiments were carried out under practical temperature conditions and the high temperature differences across the flow cross-section must influence the values obtained.

The arithmetic mean temperature difference method is very limited in its application, and it seems that its merit lies in the correlation of experimental results. In the experiments of GLASER and of LUND & DODGE, this would have been improved if the Nusselt numbers had been plotted against the Graetz number, instead of the Reynolds number.

CHAPTER VIII

CONCLUSIONS.

The particular conclusions which can be drawn from this investigation of unsteady state heat transfer in a circular tube are presented in this chapter. In general these results show none of the characteristics of results obtained by COX or JOHNSON. The next step in the continuation of this work must therefore be the investigation of the heat transfer at values of Reynolds number below 1000.

Circular Cross-section - Single Blow.

From the experiments carried out, the following conclusions apply within the range of laminar flow, $1000 < R_e < 2500$, for a circular tube .

With unsteady state heat transfer and moderate temperature gradients across the passage cross-section, the heat transfer conditions and Nusselt numbers are approximately the same for both heating and cooling, provided that,

(i) the form of wall temperature distributions are similar,

(ii) the fluid temperature profile is at no time reversed along the passage length, as a result of the simultaneous heating and cooling of the wall at different positions.

It is also shown that, once the tube has an approximately uniform temperature gradient, the Nusselt number exceeds the value of $48/11$ theoretically predicted for steady state, the difference to some extent being due to the effect of the entry length. The Nusselt number does vary until such time as this tube length temperature gradient is attained and therefore it is to be expected that because of its lower initial value, the overall value will not be as high as that for the steady state case, which has a wall temperature distribution similar to that at the end of the blow period.

If the wall temperature distribution is initially uniform, the heat transfer coefficient increases with time to a constant value which is approximately 30% above its value immediately after changeover.

The method of calculating the Nu values on a basis of arithmetic mean temperature difference has an advantage in the comparison of steady and unsteady state cases where both have uniform initial tube temperatures. For the unsteady state tests it is shown that by extrapolating to zero time, with this initial temperature distribution, the values for heating and cooling are coincident and agree with those for steady conditions, sensibly with no free convection. An advantage of this particular method is that the theoretical predicted results, with which the experimental ones agree so well, include the effect of the entry length.

Regenerators.

For counterflow regenerators with parallel uniform passages, it would appear, from theoretical considerations, and also from the experiments carried out on a single tube, that the heat transfer coefficient applicable, is approximately the value given by the theoretical calculations for steady state heat transfer in a passage with uniform temperature gradient along its length.

That these values were not obtained by COX for low values of R_e , may be a peculiarity of low Reynolds number, or may be due to the influence of the high operating temperature differences, the equivalent diameter of passage, or the reversal of gas flow in the passages.

It can be predicted that whilst eqns. (6.1) and (6.2), may give attractive values of regenerator efficiency for parallel flow operation under certain conditions, these are likely to be partly reduced by the low N_u values which would apply during the cold supply period.

In parallel flow regenerators there are times when the wall temperature distribution is uniform. In this case the $N_{u_{am}}$ value could be found from the analysis of Graetz for low values of the Graetz number or that of Lévêque for high values.

Analytical Solution.

An expression for the fluid and wall temperatures during unsteady state heat transfer, has been obtained, considering the thermal conductivity of the tube material to be zero in a direction parallel to that of the gas flow, and infinite in a direction normal to that of the gas flow. The boundary conditions required for the evaluation of this expression are,

(i) the form of variation of the gas inlet temperature with time, and

(ii) the wall temperature distribution at the origin of the unsteady state heat transfer process.

This analytical method is very suitable for determining temperatures for small values of the non-dimensional quantities, η and Λ, where the temperature boundary conditions are of the above form. The practical application is limited only to those values of the product $\eta \xi$, for which modified Bessel functions of the first kind, of zero and first order, are generally tabulated.

<u>Suggested Developments</u>.

With minor modifications to the present apparatus, further investigations relating to unsteady state heat transfer could be made.

Contra-flow operation might be arranged by reversing the connection of the cold supply blower. Several practical difficulties are foreseen in such an alteration.

Further modification would enable the investigation of unsteady state conditions in an annular passage.

Investigations at smaller Reynolds numbers will require a different design of apparatus in which the percentage heat loss will be reduced. This improvement might allow the investigation of the effect of higher temperature differences.

In all of these developments the effect of entry length will appear and the necessity of solving this problem, probably supercedes all the others.

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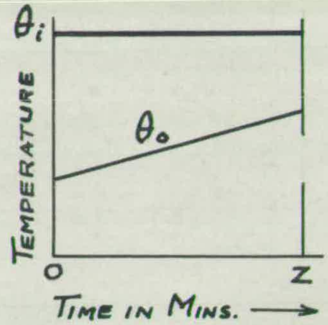
APPENDIX I

HEAT LOSS CORRECTION.



θ_i = gas inlet temperature to test section, assumed constant.

θ_o = gas outlet temperature, assumed directly proportional to time.

i.e. If a and b are constants,

$$\theta_o = az + b \quad . \tag{A1.1}$$

Total heat given up by gas,

$$Q = W\int_0^z (\theta_i - \theta_o)\,dz \quad .$$

Therefore, from equ. (A1.1),

$$Q = Wz\left(\theta_i - \tfrac{1}{2}az - b\right) \quad . \tag{A1.2}$$

In all tests the heat quantity exchanged is considered to be the heat transferred to or from the tube, say Q_T .

This heat quantity can be expressed in terms of a corrected time, ζ in an equation of the form of (A1.2), viz.

$$Q_T = W\zeta\left(\theta_i - \tfrac{1}{2}a\zeta - b\right) \tag{A1.3}$$

where ζ is the duration of time in which the gas has given up an amount of heat equal to Q_T .

Equating W in eqns. (A1.2) and (A1.3), gives

$$\zeta\left(\theta_i - \tfrac{1}{2}a\zeta - b\right) = \frac{Q_T}{Q}.z\left(\theta_i - \tfrac{1}{2}az - b\right) \quad ,$$

which can be arranged in the form,

$$(\zeta + B)^2 = \frac{1}{Q}\left[Q_T(z+B)^2 + (Q - Q_T)B^2\right] \tag{A1.4}$$

where $B = \dfrac{b - \theta_i}{a}$.

In the experimental results Q is found from the temperature records by measuring the area between the inlet and outlet temperatures. Q_T , the heat transferred to or from the matrix, is found from the difference in matrix mean temperature, which is assumed to be the mean of the two recorded values. z is the time over which Q and Q_T are measured.

With this information, equ. (Al.4) can be used to estimate ζ , the corrected time, but in fact a simplified expression was used generally.

The division of equ. (Al.4) by B^2, gives

$$\left(1 + \frac{\zeta}{B}\right)^2 = \frac{1}{Q}\left[Q_T\left(1 + \frac{z}{B}\right)^2 + Q - Q_T\right]$$

which can be expanded to the form

$$1 + \frac{2\zeta}{B} = \frac{1}{Q}\left[Q_T + Q_T \cdot \frac{2z}{B} + Q - Q_T\right] \qquad \text{(Al.5)}$$

when the terms in $\left(\zeta/B\right)^2$ and $\left(z/B\right)^2$ are neglected.

Equ. (Al.5) reduces to:

$$\zeta = \frac{Q_T}{Q} . z \qquad \text{(Al.6)}$$

This approximate relationship for corrected time is quite adequate for these experiments _ appreciable difference between the values of ζ given by equns. (Al.4) and (Al.6) only occur with higher z values.

From the measured heat quantities of periods of 2, 4, 6 and 8 minutes from the instant of changeover, these corrected times can be calculated.

In this way, a new graph of outlet temperature against corrected time can be drawn as in Fig. 10.

APPENDIX II

THEORETICAL ANALYSIS FOR GAS AND WALL TEMPERATURES IN UNSTEADY STATE CONDITIONS.

Gas and Wall Temperatures

When the following assumptions are made, the process of heat transfer between gas and wall can be analysed.

(1) The thermal conductivity of the tube material is zero in a direction parallel to that of the gas stream, and infinite in a direction at right angles to that of the gas.

(ii) The thermal properties of the gas do not vary during the process, nor does the heat transfer coefficient.

(iii) At any instant the temperature of the gas across a cross-section of the tube is constant at a mixed mean value.

(iv) There is no external heat loss.

Therefore the heat transferred to the wall is equal to the heat lost by the fluid, and per unit length,

i.e. $$hP(\theta - \tau) = -W\frac{\partial \theta}{\partial x} \qquad (A2.1)$$

Also the rate of heat transfer is equal to the rate of increase of heat content of the material,

i.e. $$hP(\theta - \tau) = M\frac{\partial \tau}{\partial z} \qquad (A2.2)$$

If z_p is the time that has elapsed since the particular supply first entered the tube, then z in the above equation, is time measured from the instant when this gas first arrived at the X-section which is being considered.

Therefore, $z = z_P - \frac{x}{v_M}$.

In most cases this term $\frac{x}{v_M}$ may be neglected as has been done here.

The boundary conditions are

$$\theta = \theta_0(z)\ ,\ x = 0 \qquad \text{(A2.3)}$$

$$\tau = \tau_0(x)\ ,\ z = 0\ . \qquad \text{(A2.4)}$$

Now writing $\xi = \frac{hPx}{W}$

and $\eta = \frac{hPz}{M}$, these equations become,

$$\theta - \tau = -\frac{\partial\theta}{\partial\xi} \qquad \text{(A2.1a)}$$

$$\theta - \tau = \frac{\partial\tau}{\partial\eta} \qquad \text{(A2.2a)}$$

where ξ and η are non-dimensional.

Now writing the expression for the double Laplace transform of θ and τ ,

$$\bar{\theta}(p,q) = \int_0^\infty\int_0^\infty \theta(\xi,\eta)\, e^{-p\xi-q\eta}\, d\xi\, d\eta \qquad \text{(A2.5)}$$

and $$\bar{\tau}(p,q) = \int_0^\infty\int_0^\infty \tau(\xi,\eta)\, e^{-p\xi-q\eta}\, d\xi\, d\eta\ , \qquad \text{(A2.6)}$$

the relationship,

$$\int_0^\infty\int_0^\infty \frac{\partial}{\partial\xi}\theta(\xi,\eta)\, e^{-p\xi-q\eta}\, d\xi\, d\eta = \int_0^\infty e^{-q\eta} d\eta \int_0^\infty \frac{\partial\theta}{\partial\xi}\, e^{-p\xi} d\xi$$

$$= \int_0^\infty e^{-q\eta} d\eta \left[\theta e^{-p\xi}\Big|_0^\infty + p\int_0^\infty \theta e^{-p\xi} d\xi\right]$$

$$= -\int_0^\infty \theta_0(\eta) e^{-q\eta} d\eta + p\int_0^\infty\int_0^\infty \theta(\xi,\eta)\, e^{-p\xi-q\eta} d\xi\, d\eta$$

$$= -\bar{\theta}_0(q) + p\,\bar{\theta}(p,q) \qquad \text{(A2.7)}$$

is found, where

$$\bar{\theta}_0(q) = \int_0^\infty \theta_0(\eta)\, e^{-q\eta}\, d\eta \quad .$$

Similarly,

$$\int_0^\infty\!\!\int_0^\infty \frac{\partial}{\partial\eta}\, \tau(\xi,\eta)\, e^{-p\xi - q\eta}\, d\xi\, d\eta = -\bar{\tau}_0(p) + q\,\bar{\tau}(p,q) \tag{A2.8}$$

where $\bar{\tau}_0(p) = \int_0^\infty \tau_0(\xi)\, e^{-p\xi}\, d\xi$.

Multiplying eqns. (A2.1a) and (A2.2a) by $e^{-p\xi - q\eta}$, and integrating with respect to ξ and η over the ranges $0 \leqslant \xi \leqslant \infty$, $0 \leqslant \eta \leqslant \infty$, and using eqns. (A2.7) and (A2.8), gives

$$\bar{\theta}(p,q) - \bar{\tau}(p,q) = \bar{\theta}_0(q) - p\,\bar{\theta}(p,q) \tag{A2.9}$$

and

$$\bar{\theta}(p,q) - \bar{\tau}(p,q) = -\bar{\tau}_0(p) + q\,\bar{\tau}(p,q) \quad . \tag{A.2.10}$$

Therefore, from (A2.9) $(1+p)\,\bar{\theta}(p,q) - \bar{\tau}(p,q) = \bar{\theta}_0(q)$

and from (A2.10) $\bar{\theta}(p,q) - (1+q)\,\bar{\tau}(p,q) = -\bar{\tau}_0(p)$.

From these two equations combined, the following expressions are obtained

$$\bar{\theta}(p,q) = \frac{(1+q)\,\bar{\theta}_0(q) + \bar{\tau}_0(p)}{pq + p + q} \tag{A2.11}$$

and

$$\bar{\tau}(p,q) = \frac{\bar{\theta}_0(q) + (1+p)\,\bar{\tau}_0(p)}{pq + p + q} \quad . \tag{A2.12}$$

The inversion theorem for double Laplace transforms gives

$$\theta(\xi,\eta) = -\frac{1}{4\pi}\int_{c_1 - i\omega}^{c_1 + i\omega}\int_{c_2 - i\omega}^{c_2 + i\omega} \bar{\theta}(p,q)\, e^{p\xi + q\eta}\, dp\, dq$$

and

$$\tau(\xi,\eta) = -\frac{1}{4\pi}\int_{c_3 - i\omega}^{c_3 + i\omega}\int_{c_4 - i\omega}^{c_4 + i\omega} \bar{\tau}(p,q)\, e^{p\xi + q\eta}\, dp\, dq$$

where C_1, C_2, C_3, and C_4 are real numbers sufficiently large to ensure the convergence of the integrals.

In this way from eqns. (A2.11) and (A2.12),

$$\theta(\xi,\eta) = -\frac{1}{4\pi}\int_{c_1-i\omega}^{c_1+i\omega}\int_{c_2-i\omega}^{c_2+i\omega}\frac{(1+q)\bar{\theta}_0(q)+\bar{\tau}_0(p)}{pq+p+q}\cdot e^{p\xi+q\eta}\,dp\,dq \qquad (A2.13)$$

and

$$\tau(\xi,\eta) = -\frac{1}{4\pi}\int_{c_3-i\omega}^{c_3+i\omega}\int_{c_4-i\omega}^{c_4+i\omega}\frac{\bar{\theta}_0(q)+(1+p)\bar{\tau}_0(p)}{pq+p+q}\cdot e^{p\xi+q\eta}\,dp\,dq \qquad (A2.14)$$

These integrals when evaluated as shown at the end of this appendix by substituting eqns. (A2.21) and (A2.24) in equ. (A2.25), give the following expressions for θ and τ.

$$\theta(\xi,\eta) = e^{-\xi-\eta}\Big[\theta_0(\eta)e^{\eta} + \sqrt{\xi}\int_0^{\eta}\frac{\theta_0(\alpha)e^{\alpha}}{\sqrt{\eta-\alpha}}\, I_1\Big\{2\sqrt{\xi(\eta-\alpha)}\Big\}\,d\alpha$$
$$+\int_0^{\xi}\tau_0(\alpha)e^{\alpha}\, I_0\Big\{2\sqrt{\eta(\xi-\alpha)}\Big\}\,d\alpha\Big] \qquad (A2.15)$$

and by symmetry,

$$\tau(\xi,\eta) = e^{-\xi-\eta}\Big[\tau_0(\xi)e^{\xi} + \sqrt{\eta}\int_0^{\xi}\frac{\tau_0(\alpha)e^{\alpha}}{\sqrt{\xi-\alpha}}\cdot I_1\Big\{2\sqrt{\eta(\xi-\alpha)}\Big\}\,d\alpha$$
$$+\int_0^{\eta}\theta_0(\alpha)e^{\alpha}\, I_0\Big\{2\sqrt{\xi(\eta-\alpha)}\Big\}\,d\alpha\Big] \qquad (A2.16)$$

$I_0(x)$ and $I_1(x)$ are modified Bessel functions of the first kind of zero and first order respectively.

It has been verified that these eqns. (A2.15) and (A2.16) along with the boundary conditions, satisfy eqns. (A2.1a) and (A2.2a).

Evaluation of $\theta(\xi,\eta)$.

In the evaluation of $\theta(\xi,\eta)$ it was found convenient to rearrange part of equ. (A2.15) as follows

$$e^{-\xi-\eta}\left[\theta_o(\eta)e^{\eta}+\sqrt{\xi}\int_0^{\eta}\frac{\theta_o(\alpha)e^{\alpha}}{\sqrt{\eta-\alpha}}I_1\left\{2\sqrt{\xi(\eta-\alpha)}\right\}d\alpha\right]$$

$$= e^{-\xi-\eta}\left[\theta_o(\eta)e^{\eta}+\sqrt{\xi}\,.\,e^{\eta}\int_0^{\eta}\frac{\theta_o(\eta-v)}{\sqrt{v}}e^{-v}I_1\left\{2\sqrt{\xi v}\right\}dv\right].$$

It is noted that, since

$$\frac{I_1\left\{2\sqrt{\xi v}\right\}}{\sqrt{v}} = \frac{1}{\sqrt{v}}\left\{\frac{\frac{1}{2}(2\sqrt{\xi v})}{1.\Gamma(2)}+\frac{\left[\frac{1}{2}(2\sqrt{\xi v})\right]^3}{1.\Gamma(3)}+\cdots\cdots\right\}$$

then $\dfrac{I_1\left\{2\sqrt{\xi v}\right\}}{\sqrt{v}} \to \sqrt{\xi}$ as $v \to 0$. (A2.17)

Evaluation of Integrals of Equations (A2.13) and (A2.14).

Eqns. (A2.13) and (A2.14) contain integrals which must be evaluated in parts.

Consider first,

$$P = -\frac{1}{4\pi}\int_{c_1-i\omega}^{c_1+i\omega}\int_{c_2-i\omega}^{c_2+i\omega}\frac{\overline{\tau}_o(p)}{pq+p+q}e^{p\xi+q\eta}\,dp\,dq$$

$$= \frac{1}{2\pi i}\int_{c_1-i\omega}^{c_1+i\omega}\overline{\tau}_o(p)\,e^{p\xi}\frac{dp}{p+1}\cdot\frac{1}{2\pi i}\int_{c_2-i\omega}^{c_2+i\omega}\frac{e^{q\eta}}{q+\frac{p}{p+1}}\,dq \qquad \text{(A2.18)}$$

now

$$\frac{1}{2\pi i}\int_{c_1-i\omega}^{c_2+i\omega}\frac{e^{q\eta}}{q+\frac{p}{p+1}} = e^{-\frac{p}{p+1}\cdot\eta}\ .$$

This is one of the tabulated Laplace transforms.

Therefore $$P = \frac{1}{2\pi i}\int_{c_1-i\omega}^{c_1+i\omega}\overline{\tau}_o(p)\,e^{\frac{-p}{p+1}\eta+p\xi}\,.\,\frac{dp}{p+1}\ . \qquad \text{(A2.19)}$$

Using the transformation $p = \frac{1}{2}\cdot\frac{S}{\xi}-1$, and noting that the line $p = c_1$ in the p-plane transforms into the line , $s = 2\xi(c_1+1) = c_s$ say, in the S-plane, part of this integral of equ. (A2.19)

$$\frac{1}{2\pi i}\int_{c_1-i\omega}^{c_1+i\omega}e^{-\frac{p}{p+1}\eta+p\xi}\,\frac{dp}{p+1} = \frac{e^{-\xi-\eta}}{2\pi i}\int_{c_s-i\omega}^{c_s+i\omega}e^{\frac{1}{2}(s+\frac{4\xi\eta}{s})}\cdot\frac{ds}{s}$$

$$= e^{-\xi-\eta}I_o(2\sqrt{\xi\eta})\ . \qquad \text{(A2.20)}$$

The inverse transform giving this result was obtained from GRAY, MATHEWS and MACROBERT.

$I_0(x) = J_0(ix)$ is the modified Bessel Function of the first kind of zero order.

Thus $\dfrac{e^{-\frac{p}{p+1}\eta}}{p+1}$ and $\bar{\tau}_0(p)$ are the Laplace Transforms of $e^{-\xi-\eta} I_0(2\sqrt{\xi\eta})$ and $\tau_0(\xi)$ respectively. The integrand of equ. (A2.19) therefore, reduces to the product of two Laplace Transforms.

From the Faltung theorem, therefore,

$$P = \int_0^{\xi} \tau_0(\alpha) e^{-(\xi-\alpha)-\eta} I_0\left(2\sqrt{\eta(\xi-\alpha)}\right) d\alpha$$

$$= e^{-\xi-\eta}\int_0^{\xi} \tau_0(\alpha) e^{\alpha} I_0\left(2\sqrt{\eta(\xi-\alpha)}\right) d\alpha \quad . \qquad \text{(A2.21)}$$

Returning to eqns. (A2.13) and (A2.14), next consider

$$Q = -\frac{1}{4\pi}\int_{c_1-i\omega}^{c_1+i\omega}\int_{c_2+i\omega}^{c_2+i\omega} \frac{\bar{\theta}_0(q)}{pq+p+q} e^{p\xi+q\eta}\, dp\, dq \quad . \qquad \text{(A2.22)}$$

From the symmetry of p and q in this integral and P, it is seen that,

$$Q = e^{-\xi-\eta}\int_0^{\eta} \theta_0(\alpha) e^{\alpha} I_0\left(2\sqrt{\xi(\eta-\alpha)}\right) d\alpha \quad . \qquad \text{(A2.23)}$$

Differentiating equ. (A2.23) under this integral sign, gives,

$$\frac{dQ}{d\eta} = -\frac{1}{4\pi}\int_{c_1-i\omega}^{c_1+i\omega}\int_{c_2-i\omega}^{c_2+i\omega} \frac{q\bar{\theta}_0(q)}{pq+p+q} e^{p\xi+q\eta}\, dp\, dq$$

$$= -e^{-\xi-\eta}\int_0^{\eta}\theta_0(\alpha)e^{\alpha} I_0\left(2\sqrt{\xi(\eta-\alpha)}\right) d\alpha + e^{-\xi-\eta}\left[\theta_0(\alpha)e^{\alpha} I_0\left(2\sqrt{\xi(\eta-\alpha)}\right)\right]_{\alpha=\eta}$$

$$+ e^{-\xi-\eta}\int_0^{\eta}\theta_0(\alpha)e^{\alpha} I_0'\left(2\sqrt{\xi(\eta-\alpha)}\right).\frac{\xi}{\sqrt{\xi(\eta-\alpha)}}\, d\alpha$$

$$= -e^{-\xi-\eta}\int_0^{\eta}\theta_0(\alpha)e^{\alpha} I_0\left\{2\sqrt{\xi(\eta-\alpha)}\right\} d\alpha \quad + e^{-\xi}\theta_0(\eta)$$

$$+ e^{-\xi-\eta}\sqrt{\xi}\int^{\eta}\theta_0(\alpha)e^{\alpha} I_1\left\{2\sqrt{\xi(\eta-\alpha)}\right\}\frac{d\alpha}{\sqrt{\eta-\alpha}} \quad . \qquad \text{(A2.24)}$$

From eqns. (A2.23) and (A2.24)

$$Q + \frac{dQ}{d\eta} = -\frac{1}{4\pi}\int_{c_1-i\omega}^{c_1+i\omega}\int_{c_2-i\omega}^{c_2+i\omega} \frac{(1+q)\bar{\theta}_0(q)}{pq+p+q}\, e^{p\xi+q\eta}\, dp\, dq$$

$$= e^{-\xi}\theta_0(\eta) + e^{-\xi-\eta}\sqrt{\xi}\int_0^{\eta}\theta_0(\alpha)e^{\alpha}I_1\left\{2\sqrt{\xi(\eta-\alpha)}\right\}\frac{d\alpha}{\sqrt{\eta-\alpha}} \quad . \tag{A2.25}$$

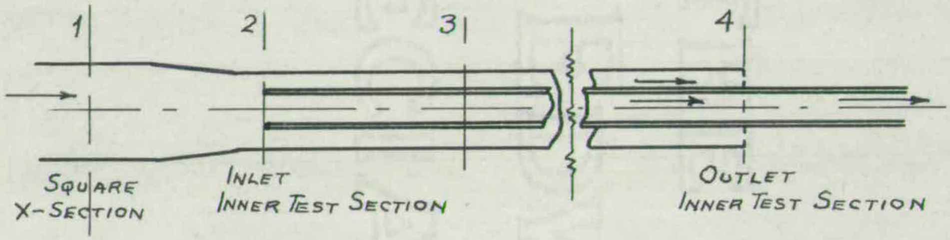
Consideration of eqns. (A2.13), (A2.18), (A2.25), shows that

$$\theta(\xi, \eta) = P + Q + \frac{dQ}{d\eta} \quad . \tag{A2.26}$$

APPENDIX III

PRESSURE DROP AT ENTRY TO TEST SECTION

In the isothermal steady flow case the entry section of the apparatus can be considered as shown .



As the leading edge of the inner test section is radiused, there is no abrupt change in flow cross section.

The cross-sectional area available for flow contracts to a minimum at 2 and remains constant for the remainder of the length. At 2, however, the flow will not have developed its parabolic velocity distribution, and this condition will not be reached until some other cross section 3.

Assuming the conservation of mechanical energy between 1 and 3,

$$\frac{K_{eb_1} v_{M_1}^{\,2}}{2g} + \frac{P_1}{\rho} = \frac{K_{eb_3} v_{M_3}^{\,2}}{2g} + \frac{P_3}{\rho} \qquad (A3.1)$$

where K_{eb} is a kinetic energy correction factor which takes into account non-uniform velocity distribution (KAYS).

$$K_{eb} = \frac{1}{A v_M^{\,3}} \int_0^A v^3 \, dA \qquad (A3.2)$$

Static pressures are assumed constant over the cross-section.

Therefore, pressure drop at entry

$$\Delta p_{1-3} = p_1 - p_3 = \rho\left(K_{eb_3}\frac{v_{M3}^2}{2g} - \frac{K_{eb_1} v_{M_1}^2}{2g}\right)$$

$$= \rho\,\frac{v_{M3}^2}{2g}\left(K_{eb_3} - K_{eb_1}\sigma^2\right) \qquad \text{(A3.3)}$$

where $\sigma = \dfrac{A_3}{A_1} = \dfrac{v_{M_1}}{v_{M3}} =$ contraction area ratio.

This pressure drop is in addition to that due to the viscosity of the gas.

Value of Kinetic Energy Correction Factor.

To evaluate equ. (A3.3), the values of the coefficients K_{eb_1} and K_{eb_3} are required.

For a circular flow cross-section with laminar flow

$$v = v_0\left(1 - \frac{r^2}{S^2}\right) \qquad \text{when} \quad r = 0,\ v = v_0 ,$$

$$\therefore K_{eb_3} = 2.00 \qquad \text{(A3.4)}$$

With steady isothermal, fully-developed laminar flow, assuming incompressible fluid, the velocity distribution at any cross section is given by the expression,

$$\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial w^2} - \frac{1}{\mu}\cdot\frac{dp}{dx} = 0 \qquad \text{(A3.5)}$$

Where $\dfrac{dp}{dx}$ can be assumed constant as in this case, the solution of the equation can be obtained for a square cross-section by relaxation methods. The resulting velocity distribution is shown in Fig. 14.

Assuming then, that in any small square

$$v(w, y) = a w^2 + b w y + c y^2 + d \qquad (A3.6)$$

where a, b, c and d are constants, the values at the mid point are obtained. With these values as the mean for each square, then

$$K_{eb_1} = \frac{1}{A v_M^3} \int_0^A v^3 dA = 1.60 \quad . \qquad (A3.7)$$

Therefore, the substitution of these values for K_{eb_3} and K_{eb_1}, in equ. (A3.3) gives the value for pressure drop at entry to the test section,

$$\Delta p = \frac{\rho v_{M_3}^2}{2g} \left(2.0 - 1.6 \sigma^2\right) \quad . \qquad (A3.8)$$

APPENDIX IV

CALCULATION OF EXPERIMENTAL HEAT TRANSFER COEFFICIENT

When this tube, with its known initial temperature distribution is either heated or cooled by a gas whose temperature is also known, the outlet gas temperature will vary in the manner of Fig. 10. Provided that the gas flow is constant, the gas temperatures can be expressed in terms of ξ and η .

Generally the temperatures used in these calculations are reduced temperatures so that the reduced gas outlet temperature is

$$\theta = \frac{\theta_o - \tau_M}{\theta_i - \tau_M}$$

where θ_o = air) gas) outlet temp.

θ_i = mean air) gas) inlet temp.

τ_M = initial mean matrix temp.

Closer investigation of the ratio η/Λ , shows that

$$\frac{\eta}{\Lambda} = \frac{W}{ML} z \quad ,$$

which is independent of the heat transfer coefficient, h . Therefore, for any point in the experimental curve of Fig. 10, the corresponding values of θ and η/Λ are known.

From the theory of Appendix II, for known boundary conditions, i.e. those of the experiment, values of θ for a range of η and ξ values can be calculated, and curves of θ against ξ for fixed values of h/Λ can be drawn. Figs. 11, 12, and 13, give sets of these curves for various boundary conditions.

The final calculation depends upon finding the ξ value on the theoretical curve for a particular h/Λ, which has the same value for θ as the experimental curve.

Once ξ i.e. Λ is known, h can be determined.

Specimen Calculations.

Test F.2. Single Blow - Hot Supply.

Weight of air/min. = 0.0472 lbs.

Air flow rate, G = 0.577 lbs/ft^2.sec.

Mean tube initial temp. = 79°F.

Mean air inlet temp. (By Simpson's Rule) = 165°F.

$\therefore$ Reduced temperature $= \dfrac{\theta - 79}{165-79} = \dfrac{\theta - 79}{86}$.

Recorded temperatures in °F are shown in Fig. 6. For corresponding reduced inlet and outlet temperatures see Fig. 9. (This outlet temp. has not yet been corrected for heat losses).

Mean temp. for property values = 132°F.

$\therefore \mu = 1.31 \times 10^{-5}$ lb/ft.sec.

and $R_e = \dfrac{Gd}{\mu} = \dfrac{0.577 \times \frac{0.5}{12}}{1.31 \times 10^{-5}} = 1840$.

X-sectional area of tube which is heated from inner passage

$= \dfrac{\pi}{4}(0.564^2 - 0.5^2)$

$= 0.0535$ ins^2 .

∴ Total wt. of tube heated from inner passage

$= 72 \times 0.0535 \times 0.323$

$= 1.242$ lbs.

∴ Water equivalent of inner portion of tube

$= 1.242 \times 0.095 = 0.1181$ B.Th.U/°F.

$= M \times L.$

The following heat quantities in B.Th.U. are found from the actual test record, by planimetering etc.

	MINUTES			
Duration of time from changeover	2	4	6	8
Heat to tube in total time Q_T	1.65	3.07	4.14	4.96
Heat from gas in total time Q	1.69	3.29	4.72	6.04
∴ Heat loss in total time	0.04	0.22	0.58	1.08
Heat loss as % of heat to tube	2.4	7.2	14.0	21.8

Assume a heat transfer coefficient, $h' = 38.6 \times 10^{-7}$ B.ThU/sec.in^2 °F.

$$\Lambda = \frac{hPL}{W} = \frac{h.\ PL}{GA.\ c_p}$$

Total surface area $PL = \pi \times 0.5 \times 72 = 113$ in^2.

$GA = 0.0472$ lbs/min. $= 0.000787$ lbs/sec.

$c_p = 0.24$ B.Th.U/lb.

∴ Assumed non-dimensional length is

$$\Lambda' = \frac{38.6 \times 10^{-7} \times 113}{0.000787 \times 0.24} = \underline{2.31}$$

Non dimensional time, $\eta = \dfrac{hP_z}{M}$,

∴ for assumed non-dimensional time, $\underline{\eta' = 1}$,

$$z' = \frac{M}{hP} = \frac{0.1181}{72} \times \frac{1}{38.6 \times 10^{-7} \times \pi \times 0.5} = \underline{4.52 \text{mins.}}$$

For heat loss correction, from equ. (A1.6), the corrected time, $\zeta = \frac{Q_T}{Q} z$.

$\therefore$ at 2 mins. $\zeta' = \frac{1.65}{1.69} \times 2 = 1.95$ mins.

and similarly, for $z' = 2 \quad 4 \quad 6 \quad 8$ mins.

corrected time, $\zeta' = 1.95 \quad 3.73 \quad 5.25 \quad 6.58$ mins.

From these values a corrected outlet temperature curve can be redrawn as shown in Fig. 10.

A table of values of θ corresponding to different ζ/Λ values, can be drawn up from Fig. 10, since $\Lambda' = 2.31$. Comparison of these θ values with those of the calculated curves of Fig. 11, will give values of Λ which contain the experimental heat transfer coefficient and not the assumed h'.

$\frac{\zeta'}{\Lambda'} = \frac{\zeta}{\Lambda}$	ζ' ($\Lambda' = 2.31$)	Time in Mins. ($4.52\zeta'$)	θ outlet (from corrected curve)	Λ from Fig.13	Λ without heat loss correction
0.7	1.61	7.3	0.42	3.35	5.6
0.6	1.38	6.25	0.362	3.3	4.65
0.5	1.15	5.21	0.300	3.4	4.0
0.4	1.92	4.17	0.250	3.2	3.45
0.3	0.693	3.13	0.205.	3.0	3.08
0.2	0.462	2.09	0.167	2.8	2.82
0.1	0.231	1.05	0.130	2.55	2.57

From a curve of Λ against η/Λ, the curve is extrapolated to the value at $\eta = 0$.

In this case, $\underline{\Lambda = 2.3.}$

$$\therefore h = \frac{2.3}{2.31} \times 38.6 \times 10^{-7} = 38.4 \times 10^{-7} \text{ B.Th.U/sec.in}^2\ {}^{o}\text{F.}$$

Since $k = 4.60 \times 10^{-6}$ B. Th.U/sec. ft.^{o}F.

$$Nu = \frac{hd}{k} = \frac{38.4 \times 10^{-7} \times 0.5 \times 12}{4.60 \times 10^{-6}} = \underline{\underline{5.0}}$$

At $\underline{\eta/\Lambda = 0.3}$, $\Lambda = 3.0$ and $\therefore$ $\underline{Nu = 6.5}$

This procedure is exactly the same in the case of cold supply, the only difference arises in the choice of theoretical curves.

In the particular example chosen here, the inlet temperature varies only slightly at the beginning. This lowers the Nu value slightly to give $Nu = 4.8$, at $\eta = 0$.

The danger of extrapolation can be offset to a certain extent by applying equ. (4. 15) directly to the temperature at $\eta = 0$. This has been used in every case to check the extrapolated value of Λ at $\eta = 0$.

APPENDIX V

FORM OF NUSSELT NUMBER FOR UNSTEADY STATE HEAT TRANSFER.

Dimensional analysis of the steady state, for h, the heat transfer coefficient based on the local mean temperature difference, gives

$$h = \frac{k}{d} \cdot f(R_e, P_r)$$

$$\text{or} \qquad N_u = f(R_e, P_r) \quad .$$

In the unsteady state case, the water equivalent of the gas and wall will influence the conduction of heat. These two quantities are already combined in the non-dimensional ratio

$$\frac{\eta}{\Lambda} = \frac{Wz}{ML} \quad .$$

To allow for the fact that in this treatment the N_u value used is that which applied to the whole length and yet is assumed constant at each particular X-section the ratio L/d will need to be included - this ratio will also bring the entry length effect within the analysis.

$$\therefore \; N_u = f\left(R_e, P_r, \eta/\Lambda, L/d\right) \quad . \qquad \text{(A5.1)}$$

The initial condition of wall temperature distribution may be included as a function of the form

$$\left(\frac{\theta_i - \tau_M}{\theta_i - \tau_i}\right) ,$$

assuming a linear temperature distribution.

θ_i = supply inlet temp.

τ_i = tube initial temp. at inlet.

τ_M = tube initial mean temp.

On the other hand it may be introduced as a coefficient having a different value for different initial wall temperature distributions. These temperatures distributions may be classified as

(i) +ve linear temp. gradient

(ii) -ve linear temp. gradient

(iii) uniform temperature.

A positive gradient is one where, for $Z > 0$, the gas-wall temperature difference has always the same sign along the length of the heat transfer surface.

A negative gradient is one in which for $Z > 0$, the temperature difference may have different signs along the length.

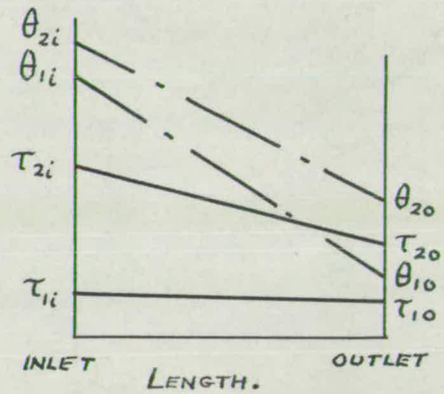
APPENDIX VI

ARITHMETIC MEAN TEMPERATURE DIFFERENCE.

Common in steady state analysis, this mean temperature difference in unsteady state, implies a position mean of temperature values which are already means with respect to time.

It is felt that this is justified by the sensibly linear variation of temperature with time in the experiments carried out.

The temperatures of the gas and the tube wall at different positions are shown alongside.



Taking time mean values first,

initial tube temp, $\tau_1 = \frac{1}{2}(\tau_{1i} + \tau_{10})$

final tube temp., $\tau_2 = \frac{1}{2}(\tau_{2i} + \tau_{20})$

gas initial temp., $\theta_1 = \frac{1}{2}(\theta_{1i} + \theta_{10})$

gas final temp., $\theta_2 = \frac{1}{2}(\theta_{2i} + \theta_{20})$.

Overall mean gas temp., $\theta_H = \frac{1}{2}(\theta_1 + \theta_2)$. (A6.1)

Total heat to tube = total heat from gas = Q

$$\therefore ML(\tau_2 - \tau_1) = Wz\left[\theta_1 - \frac{1}{2}(\theta_{10} + \theta_{20})\right] = Q \qquad (A6.2)$$

Arithmetic mean temperature difference,

$$(\Delta\theta)_{am} = \frac{1}{2}\left[(\theta_1 - \tau_1) + (\theta_2 - \tau_2)\right]$$
$$= \frac{1}{2}\left[\theta_1 + \theta_2 - 2\tau_1 - (\tau_2 - \tau_1)\right] .$$

From equns. (A6.1) and (A6.2), $(\Delta\theta)_{am} = \left(\theta_H - \tau_1 - \frac{Q}{2ML}\right)$. (A6.3)

Therefore, heat transfer coefficient based on this temperature difference,

$$h_{am} = \frac{Q}{(\Delta\theta)_{am} PLz} = \frac{Q}{\left(\theta_H - \tau_1 - \frac{Q}{2ML}\right) PLZ}$$

$$\therefore \frac{1}{h_{am}} = \left[\frac{\theta_H - \tau_1}{Q} - \frac{1}{2ML}\right] PLz \quad . \qquad \text{(A6.4)}$$

Therefore for single blow operation, in addition to gas temperatures it is only necessary to be able to estimate the initial tube mean temperature.

Where a heat loss correction is required, as in the experiments detailed here, the expression used is

$$\frac{1}{h_{am}} = \left[\frac{\theta_H' - \tau_1}{Q_T} - \frac{1}{2ML}\right] PL\,\zeta \qquad \text{(A6.5)}$$

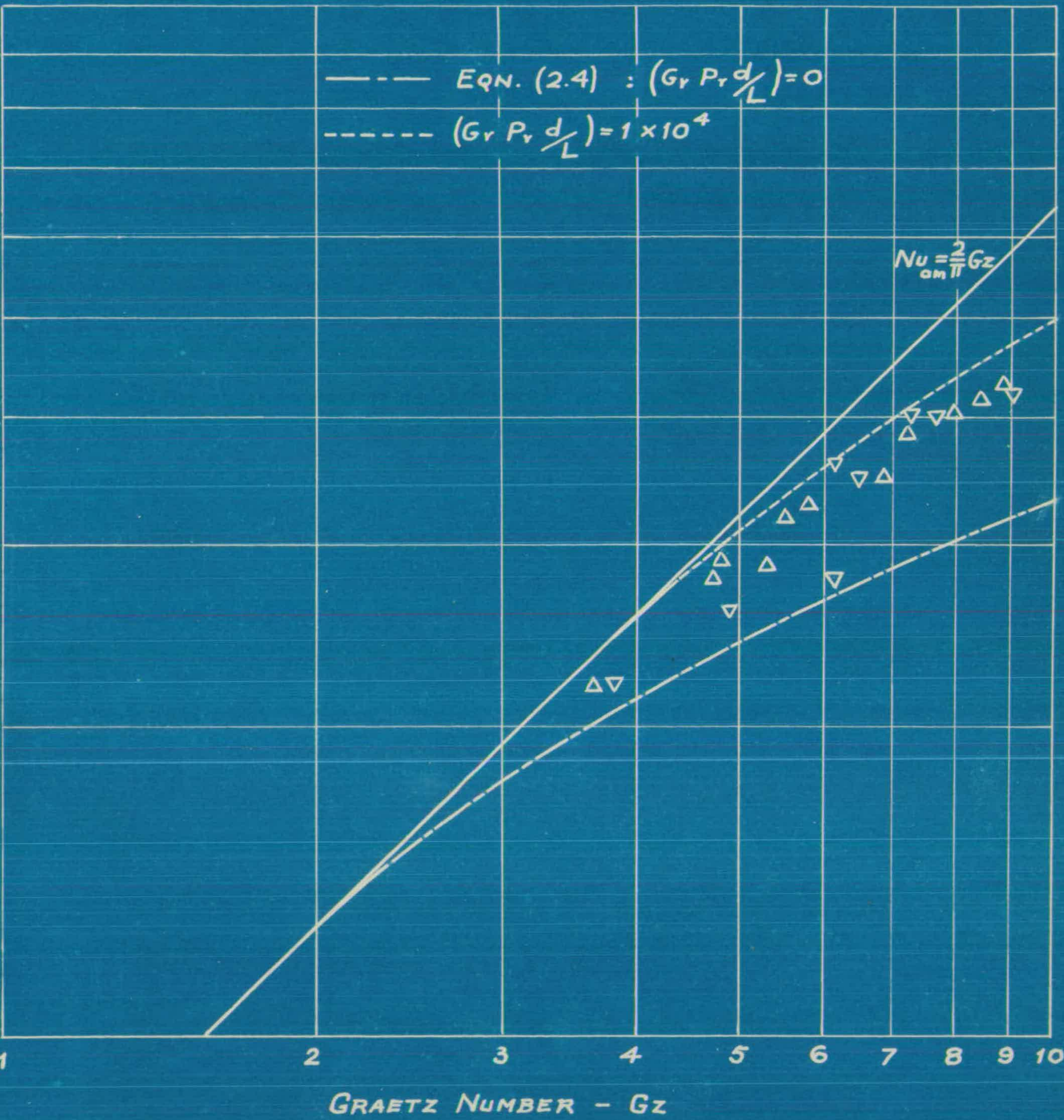
where θ_H' = overall mean gas temp. between $Z = 0$, and $Z = \zeta$.

A similar relationship can be applied to a cold supply period.

The relation for heat exchanged in cyclic operation of a regenerator, equ. (6.10)

$$Q = \frac{1}{\frac{1}{h_H z_H} + \frac{1}{h_c z_c}} \cdot PL\,(\theta_H - \theta_c)$$

can be derived from equ. (A6.5)



UNSTEADY STATE HEAT TRANSFER, $\eta = 0$
UNIFORM INITIAL TUBE TEMPERATURE

△ – GAS BLOW
▽ – AIR BLOW.

FIG. 1. VALUES OF Nu_{am}.

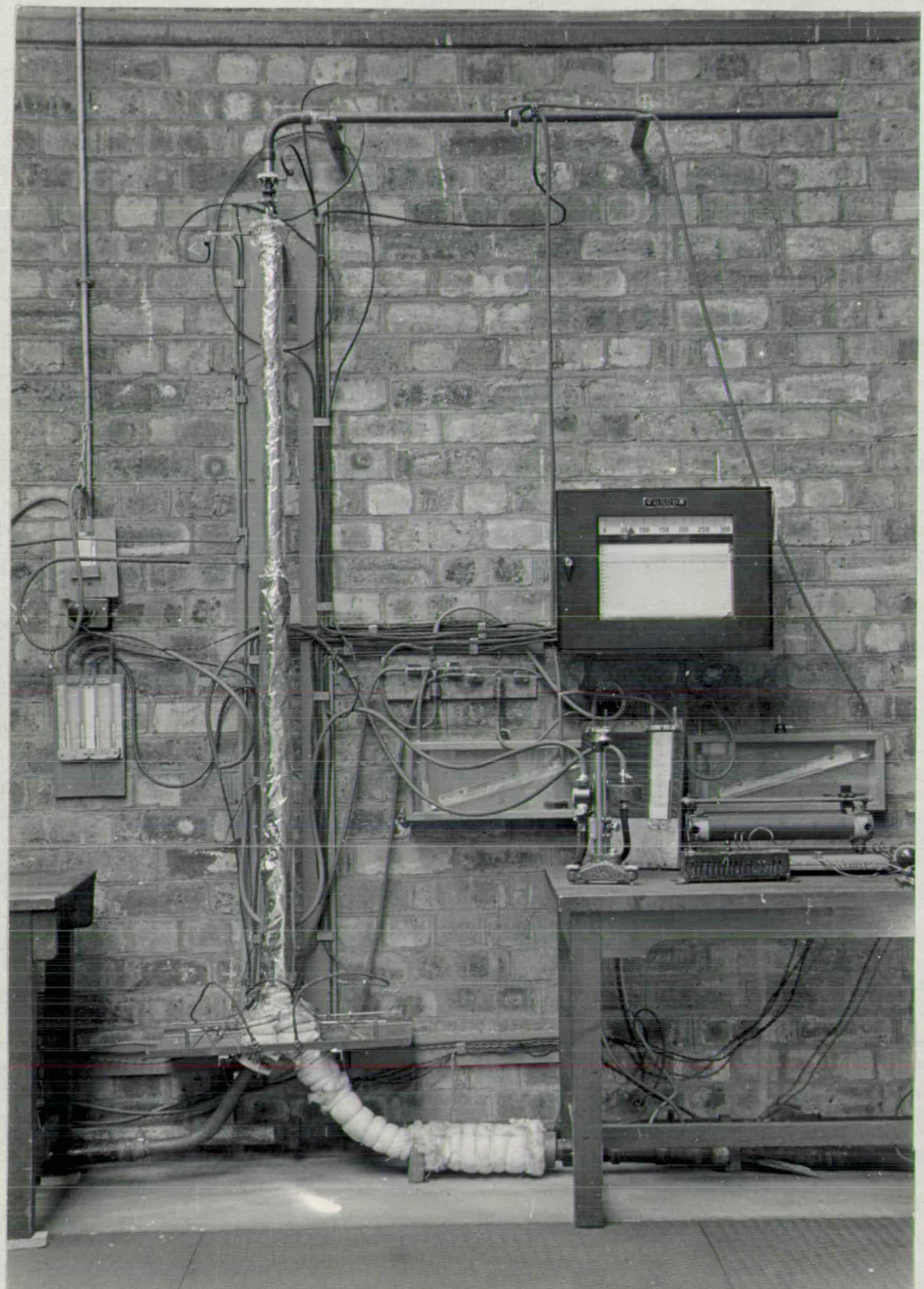


FIG. 2. GENERAL ARRANGEMENT.

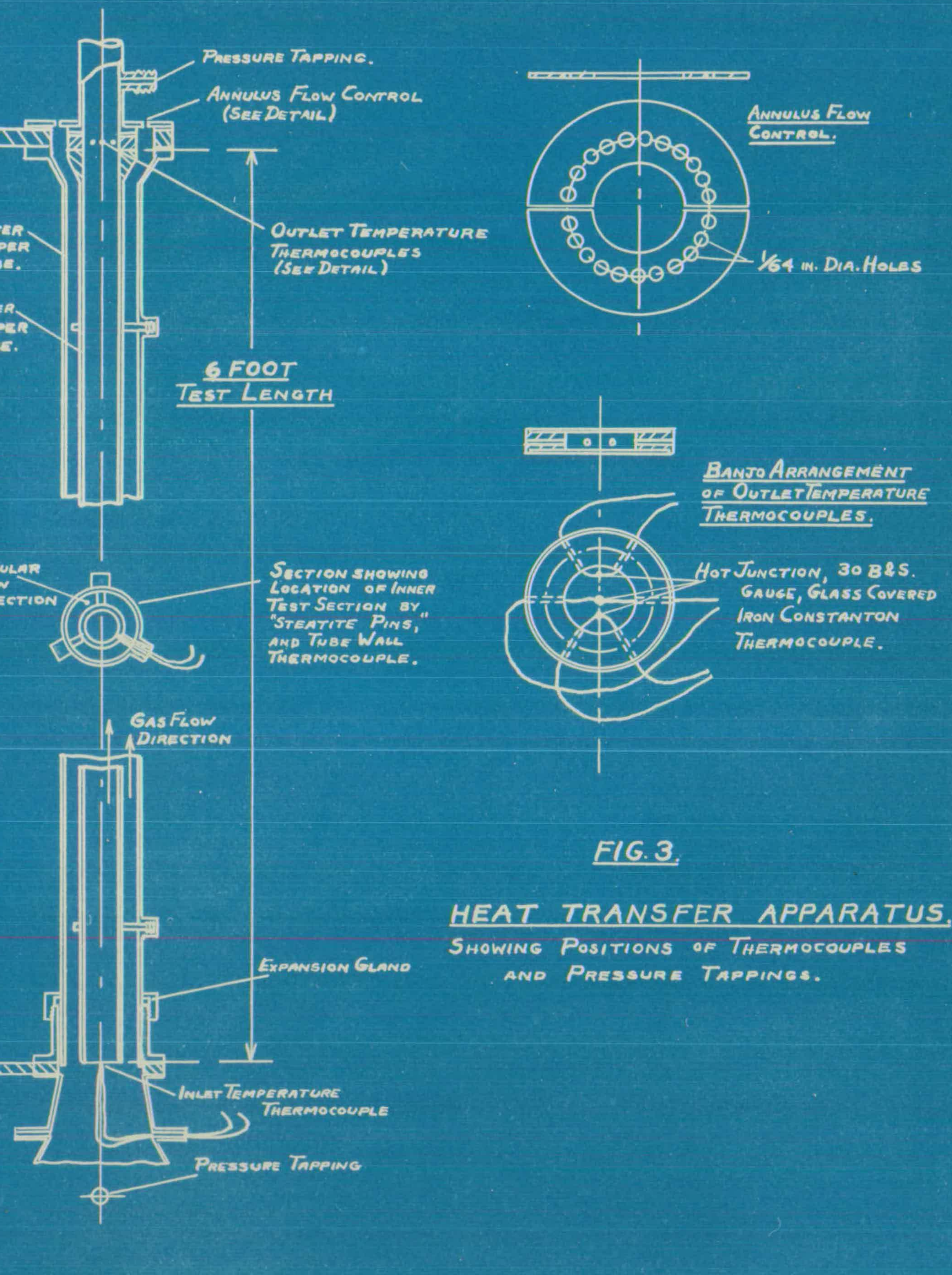
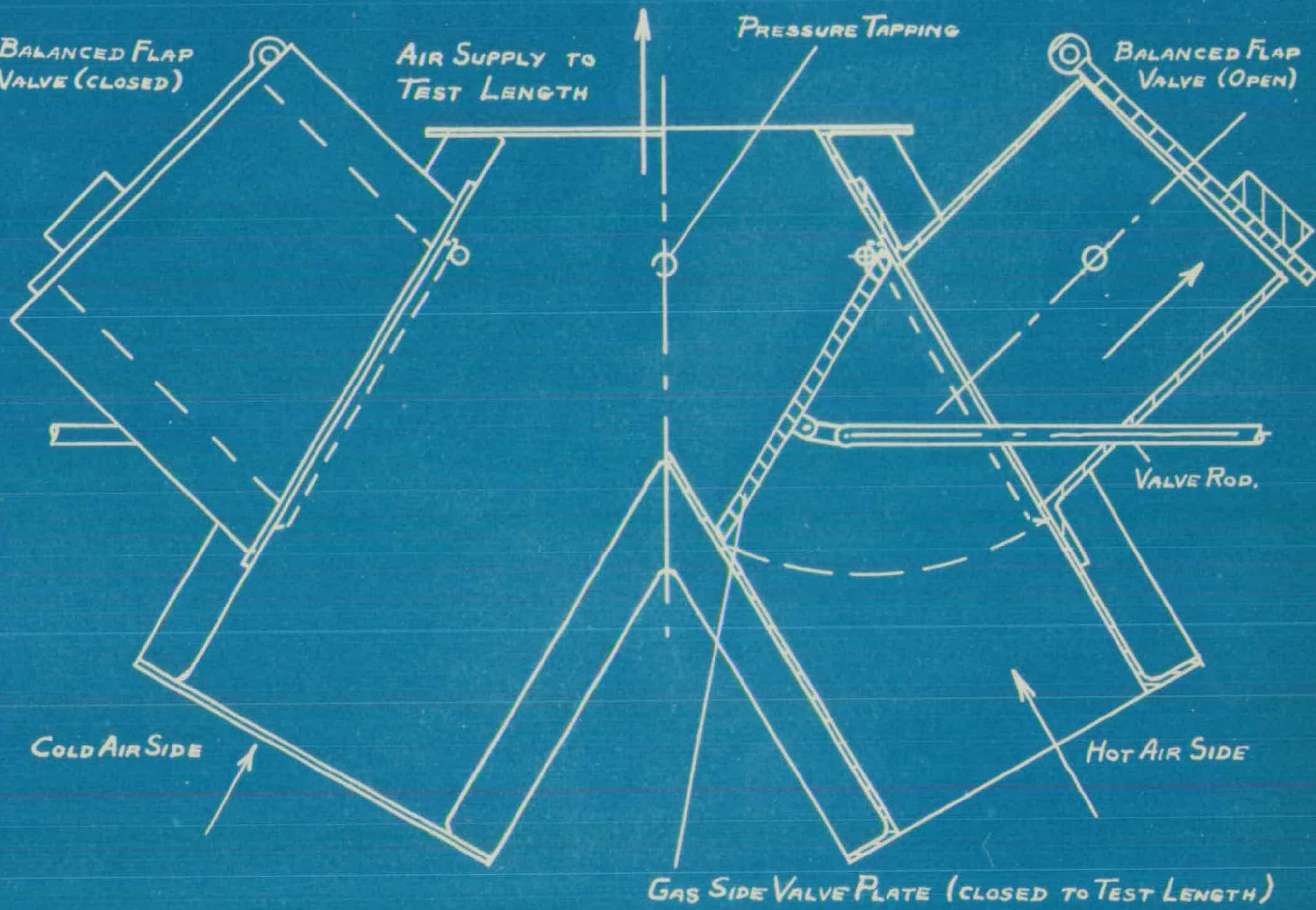


FIG. 3.

HEAT TRANSFER APPARATUS.

SHOWING POSITIONS OF THERMOCOUPLES AND PRESSURE TAPPINGS.



SELECTOR VALVE BOX.

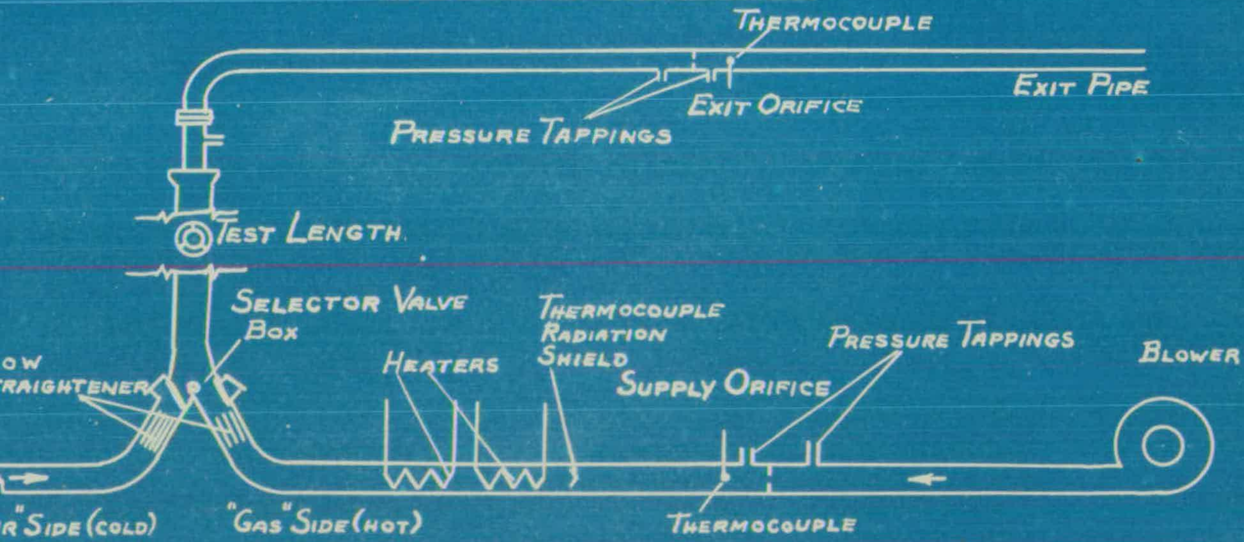


FIG. 4. FLUID SUPPLY.

ARRANGEMENT AND MEASUREMENT

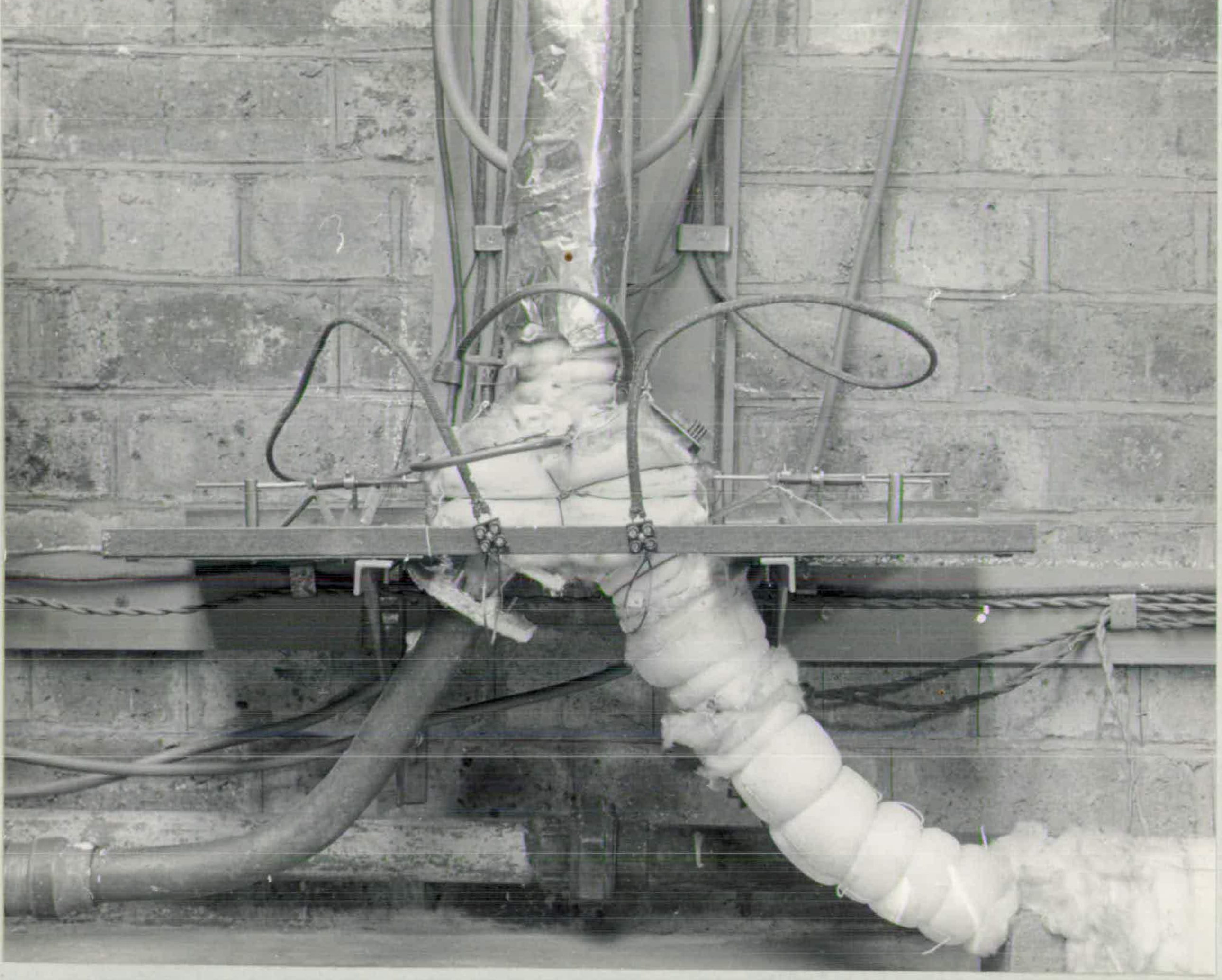


FIG. 5. SELECTOR VALVE BOX.

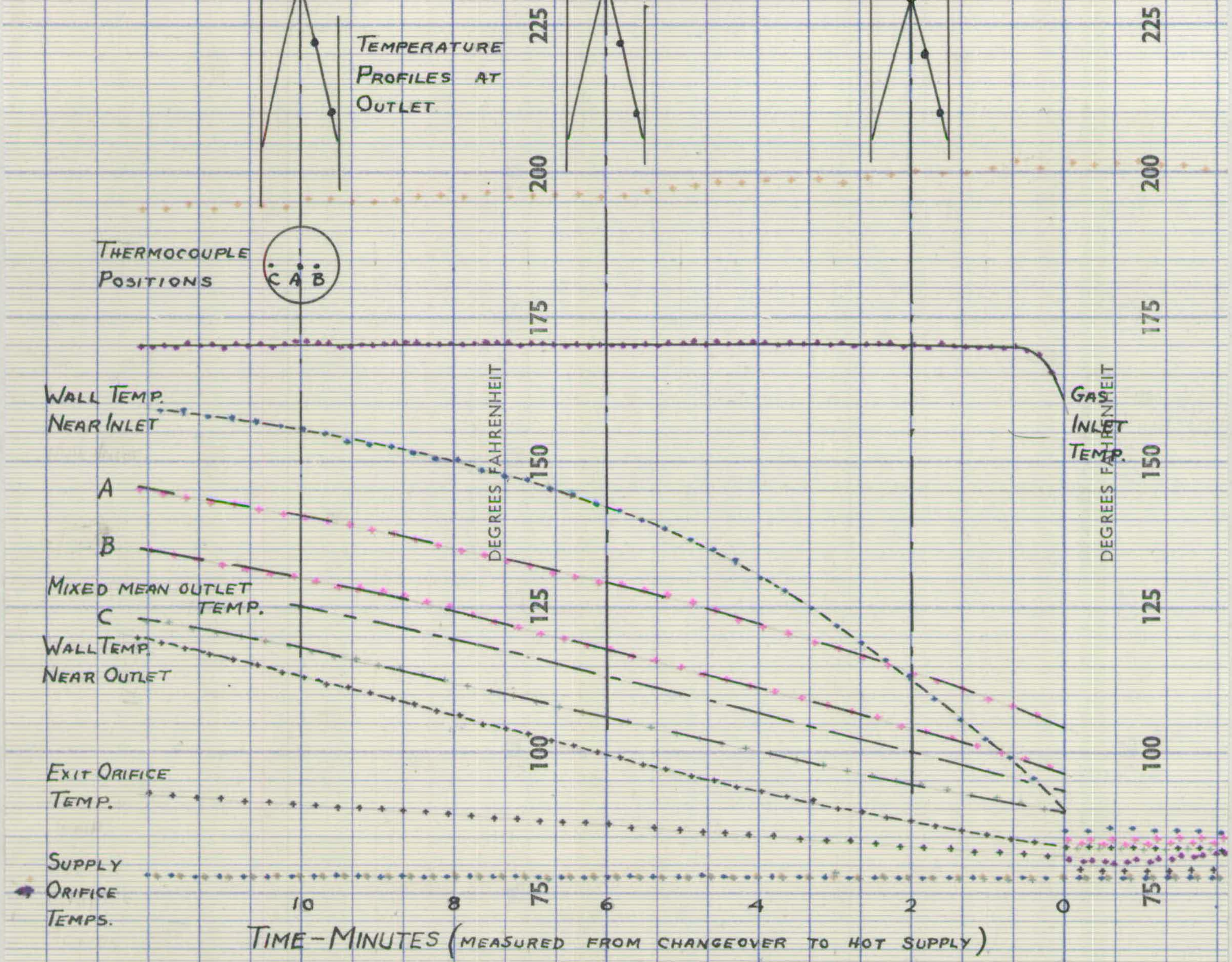


FIG. 6. ACTUAL TEMPERATURE RECORD.

SINGLE GAS BLOW, $R_e = 1960$, TEST F3.

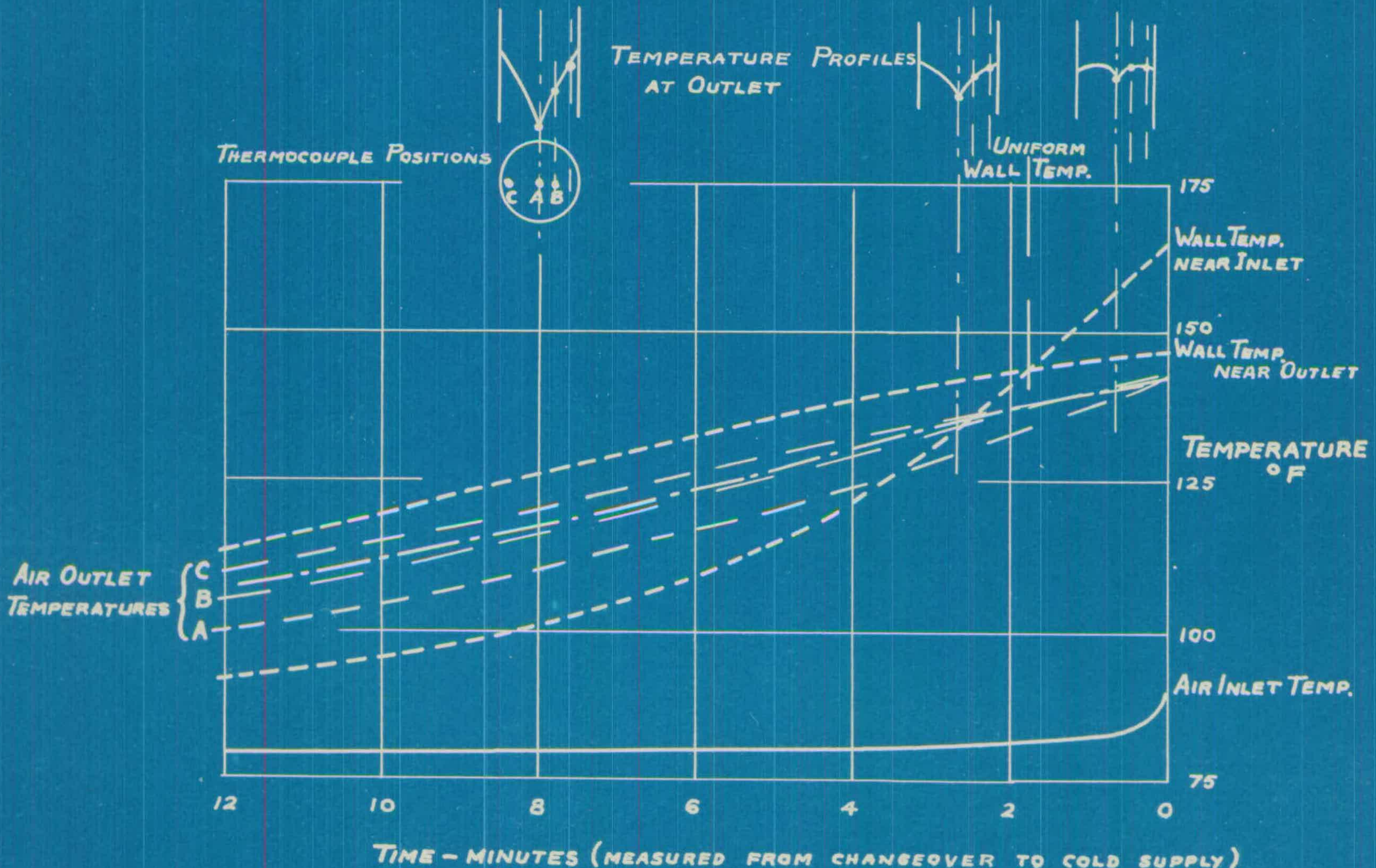


FIG. 7. TEMPERATURE RECORD.

Single Air Blow, R_e = 1960, Test G2.

—·— Mixed Mean Air Outlet Temperature

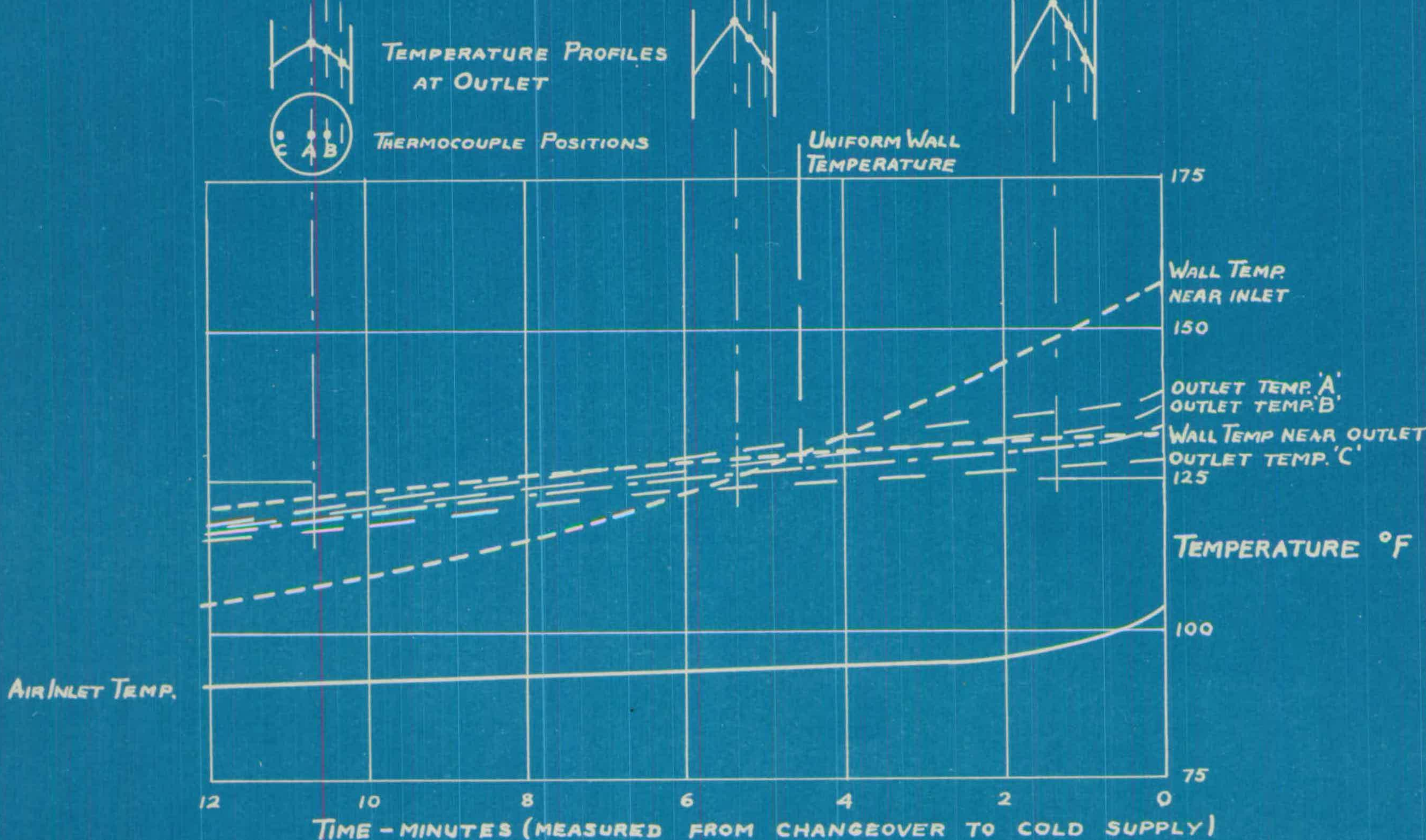


FIG. 8. TEMPERATURE RECORD.

SINGLE AIR BLOW, $R_e = 1030$, TEST G 8.

— - — MIXED MEAN AIR OUTLET TEMPERATURE

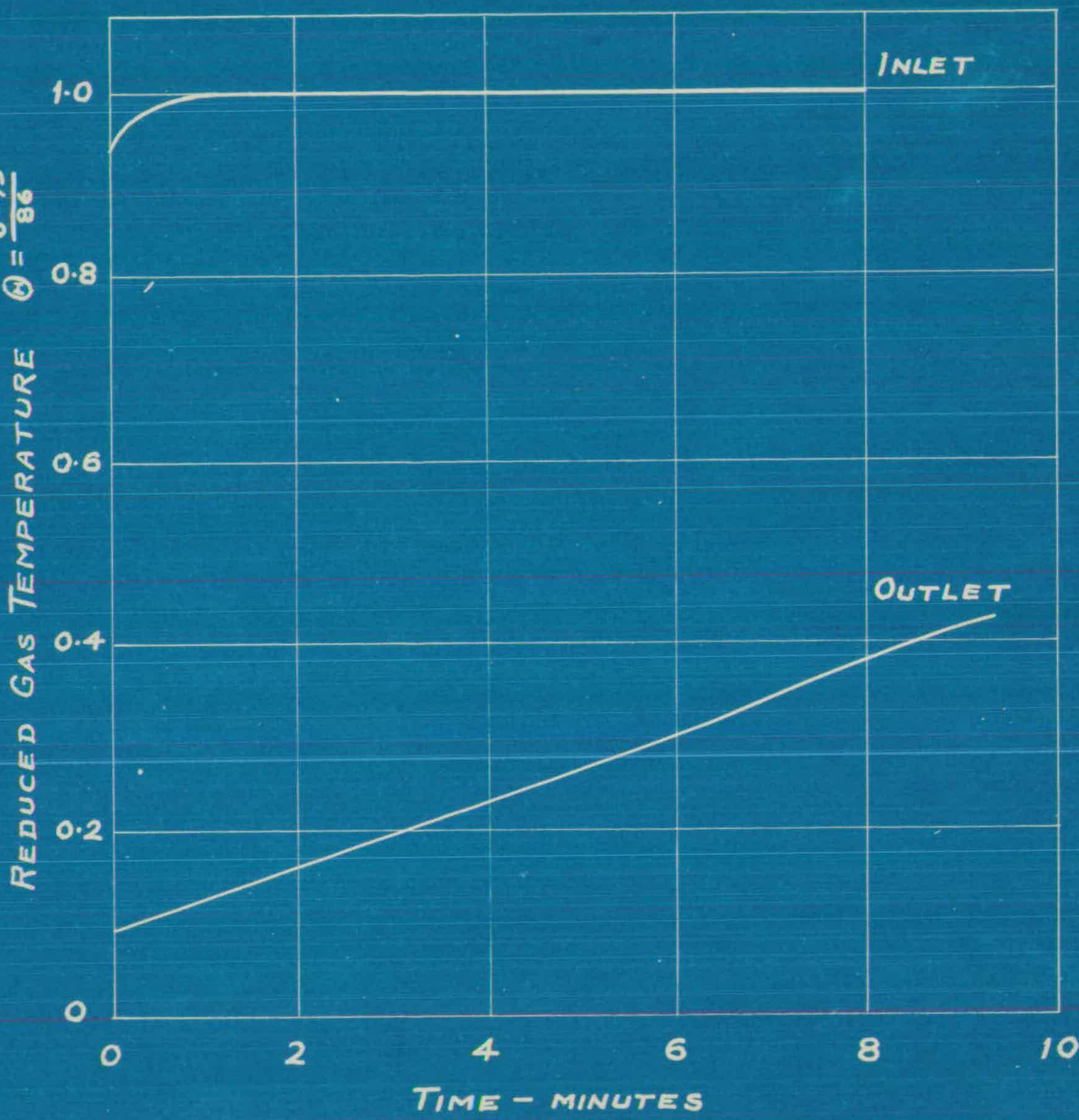


FIG. 9. REDUCED GAS TEMPERATURES.

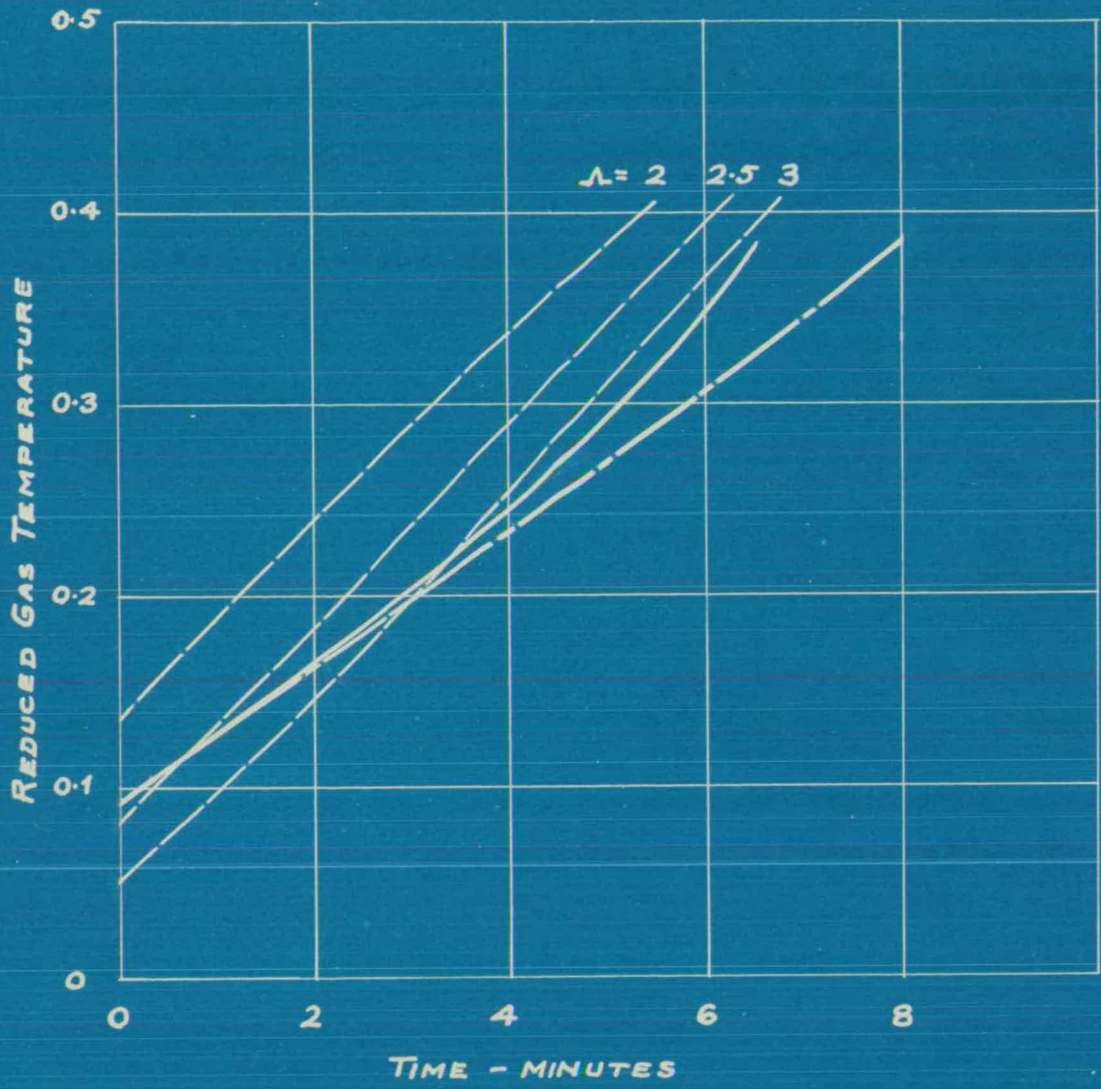


FIG. 10. GAS OUTLET TEMPERATURE.

—— - —— EXPERIMENTAL RECORD

———— CORRECTED FOR HEAT LOSSES

— — — CALCULATED VALUES $[\theta_0(\eta) = \text{const.}]$

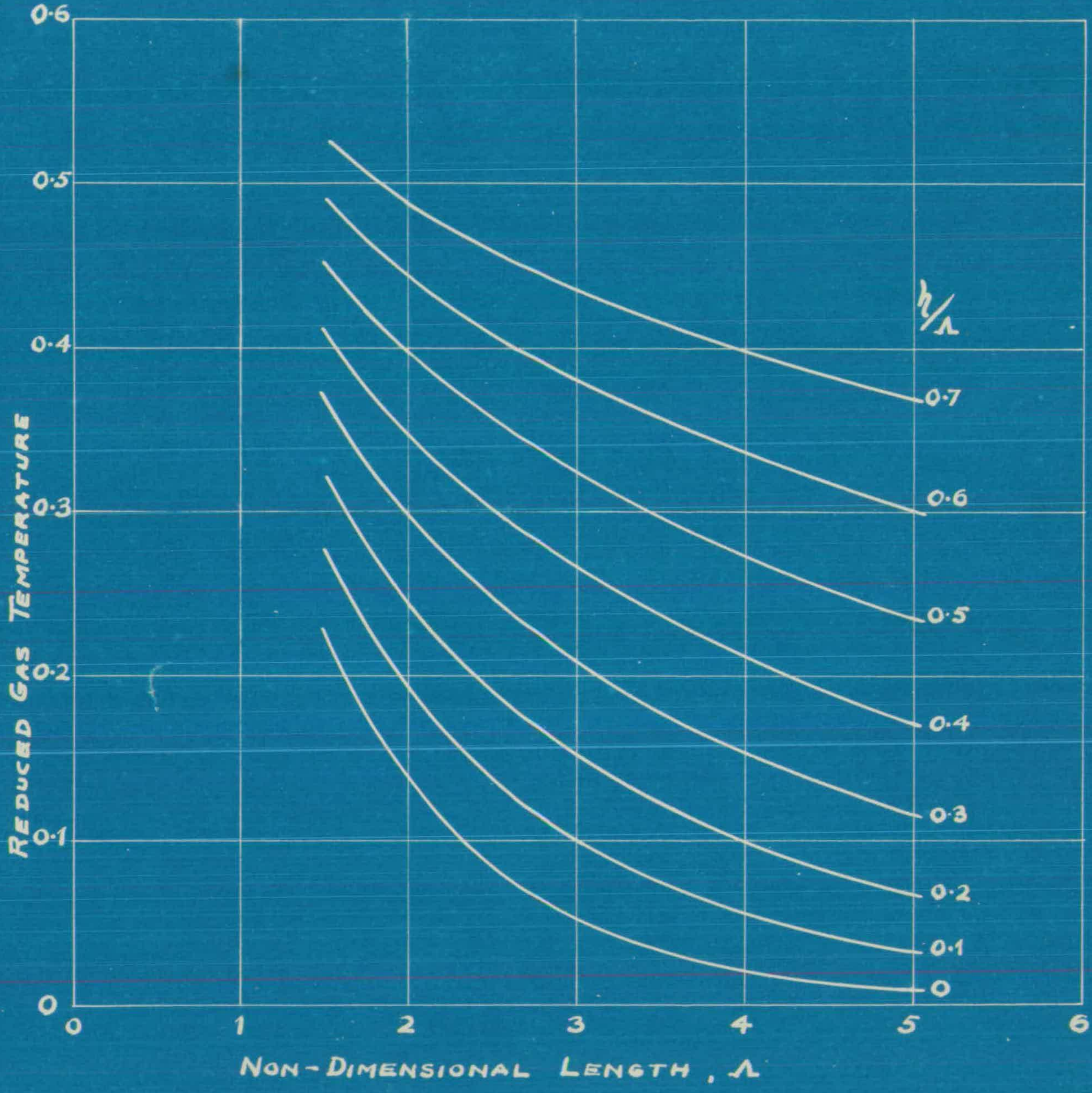


FIG. 11. CALCULATED GAS TEMPERATURE.

Constant Gas Inlet Temperature = 1

Uniform Tube Initial Temperature = 0.

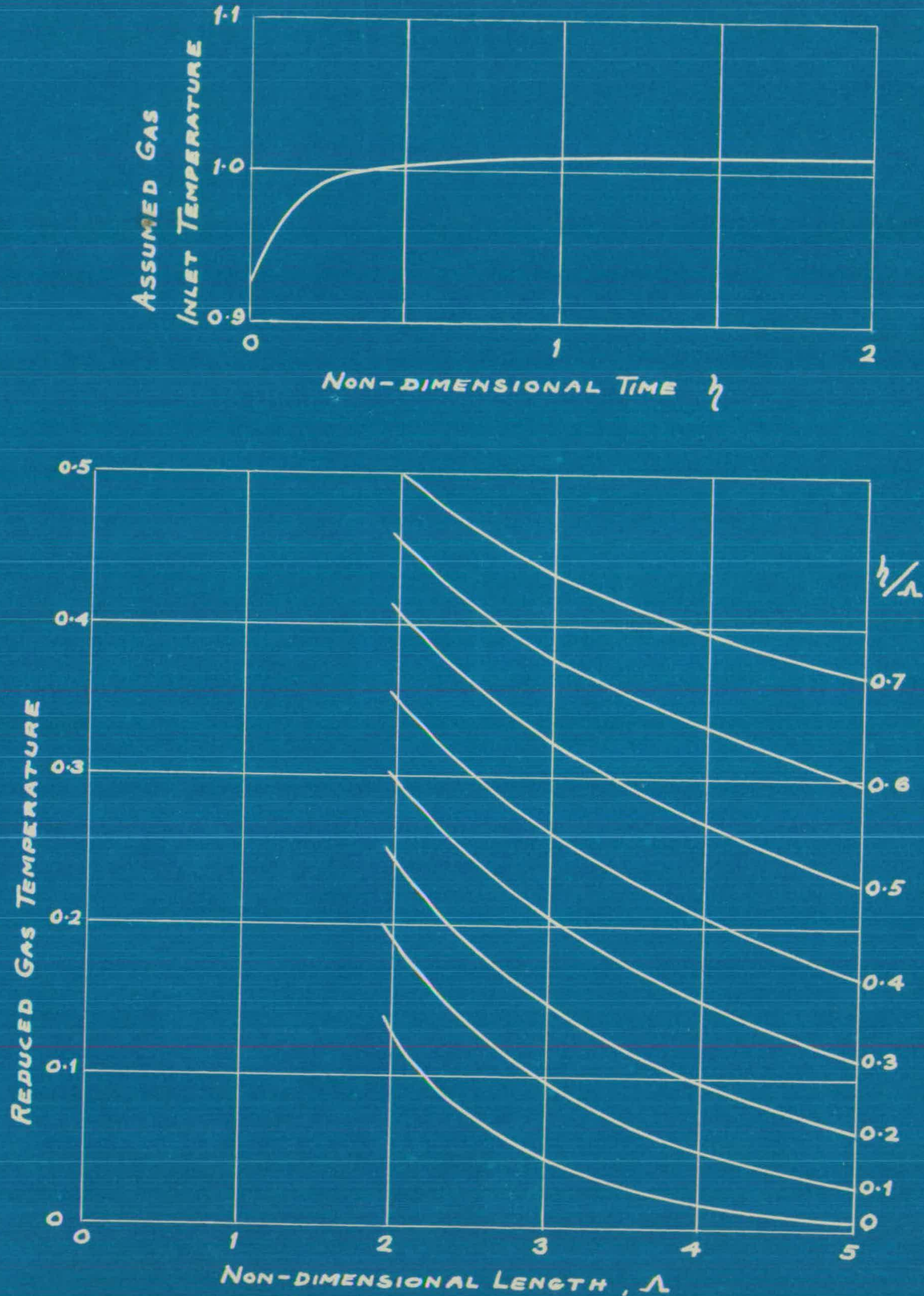


FIG. 12. CALCULATED GAS TEMPERATURE.

Varying Gas Inlet Temperature – as above

Uniform Tube Initial Temperature = 0

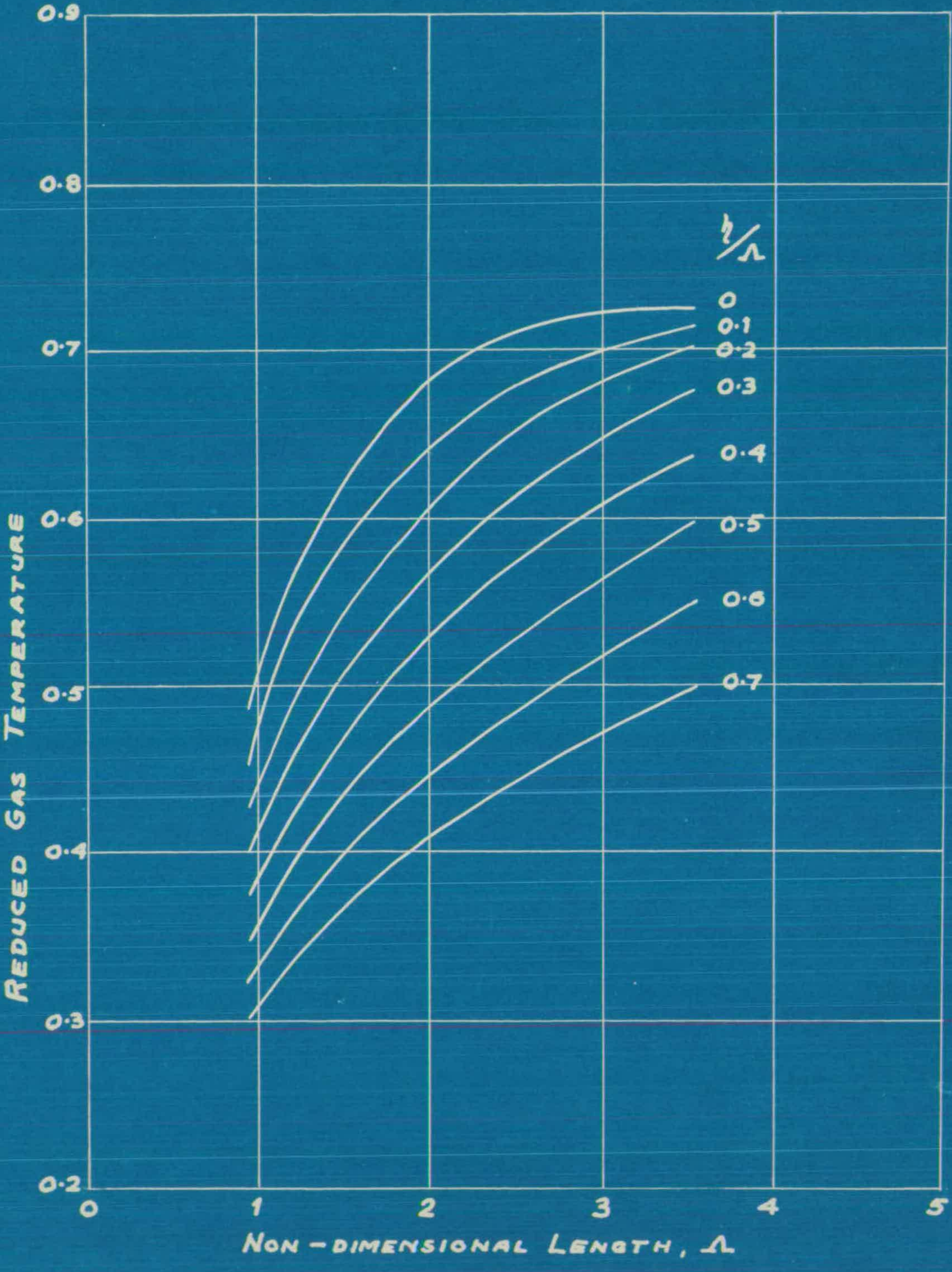


FIG. 13. CALCULATED GAS TEMPERATURE.

CONSTANT GAS INLET TEMPERATURE = 0

TUBE INITIAL TEMPERATURE, $\tau_0(\xi) = 1{\cdot}31 - \frac{0{\cdot}62}{\Lambda}\xi$

0	0	0	0	0
0·84	0·81	0·69	0·44	0
1·39	1·32	1·10	0·69	0
1·69	1·60	1·32	0·81	0
1·78	1·69	1·39	0·84	0

℄ ℄

0·63	0·60	0·52	0·33
1·24	1·16	0·93	0·52
1·59	1·48	1·16	0·60
1·74	1·59	1·24	0·63

℄ ℄

FIG. 14. VELOCITY DISTRIBUTION.

LAMINAR FLOW – SQUARE CROSS-SECTION

$v_M = 1$

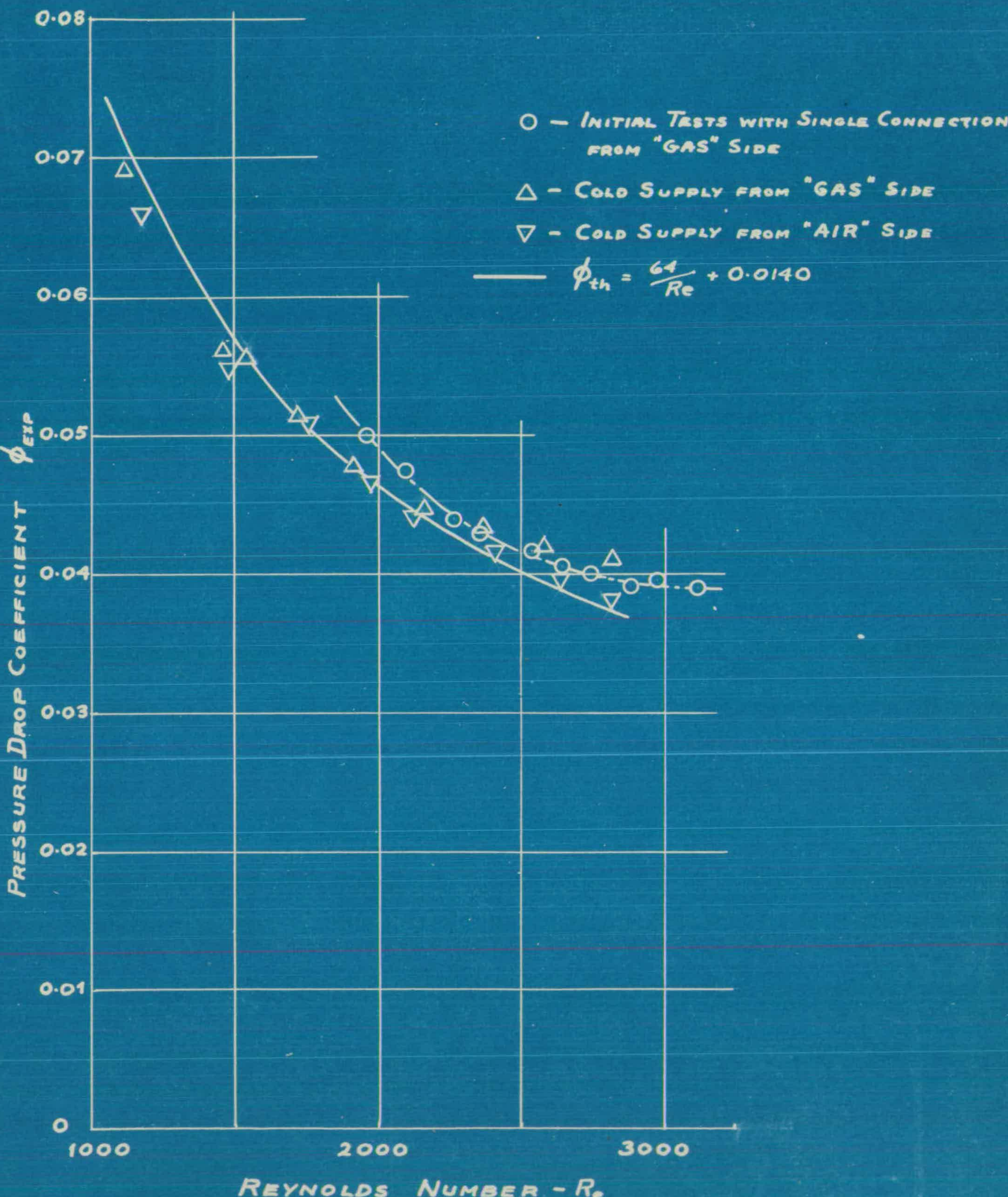


FIG. 15. PRESSURE DROP COEFFICIENTS.

STEADY STATE — NO HEAT TRANSFER

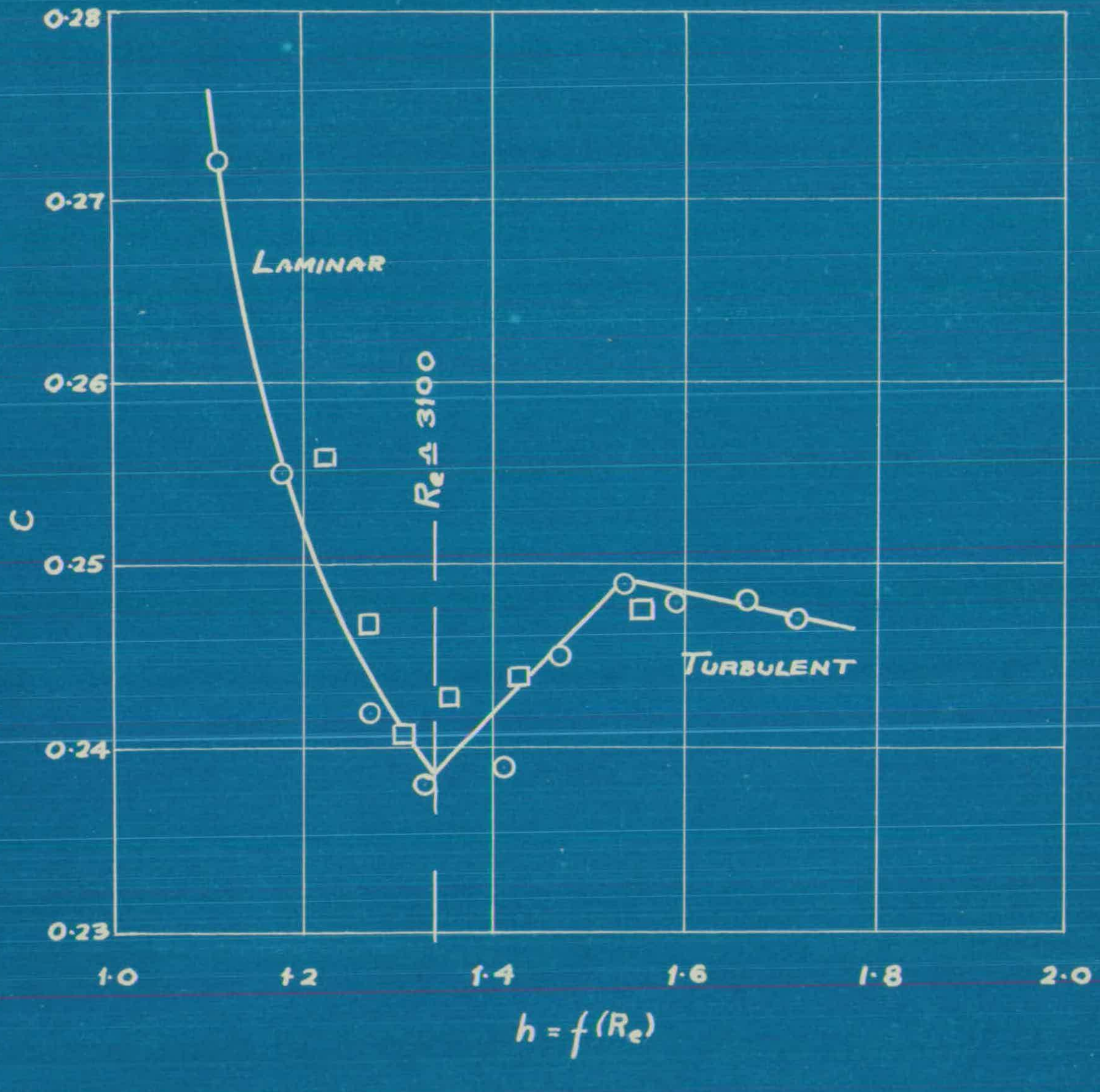


FIG. 16. PRESSURE DROP COEFFICIENT, "C".

STEADY STATE - NO HEAT TRANSFER

(ESTIMATED FROM FLOW AT SUPPLY ORIFICE)

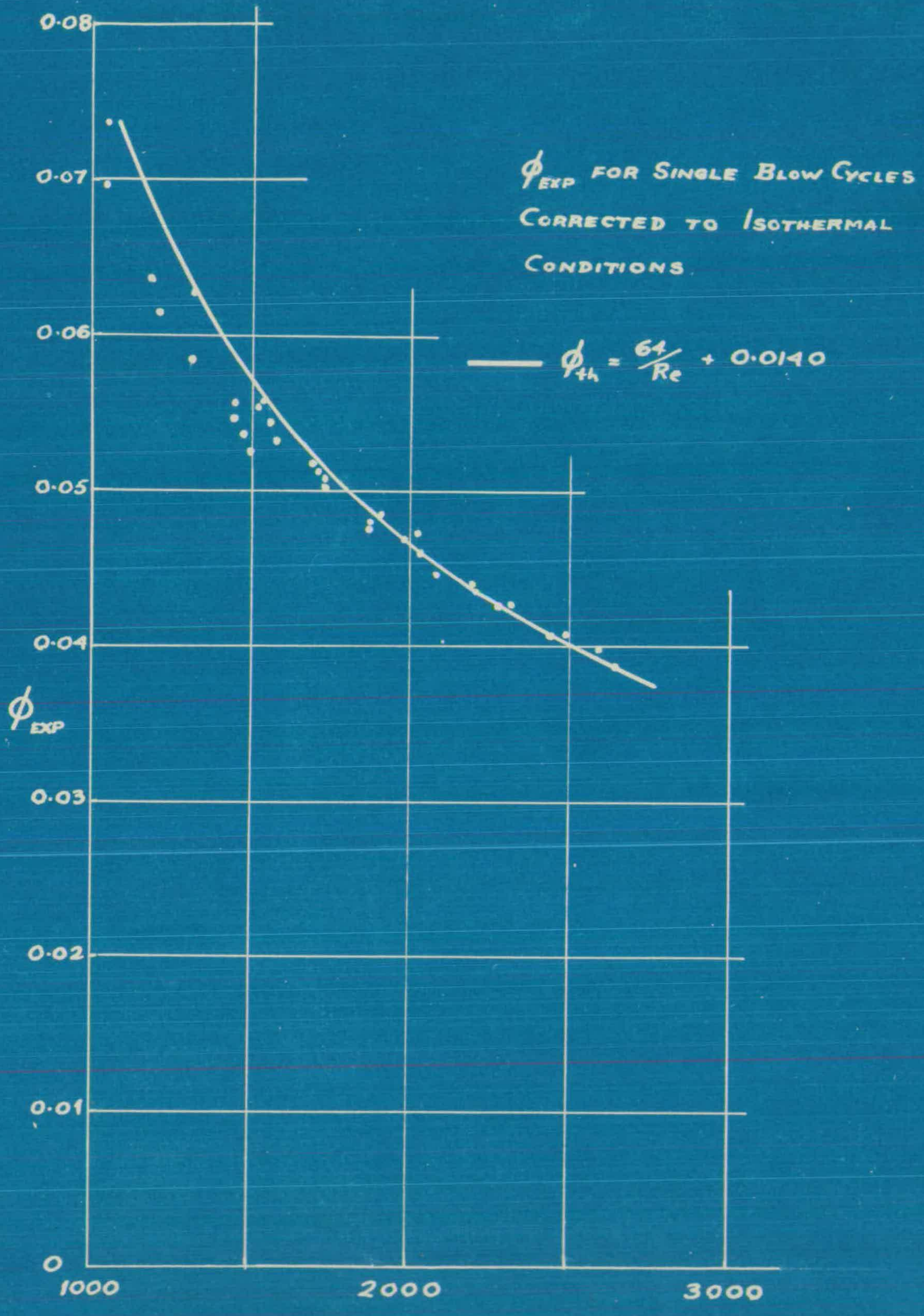


FIG. 17. PRESSURE DROP COEFFICIENTS.

UNSTEADY STATE – AIR AND GAS BLOW

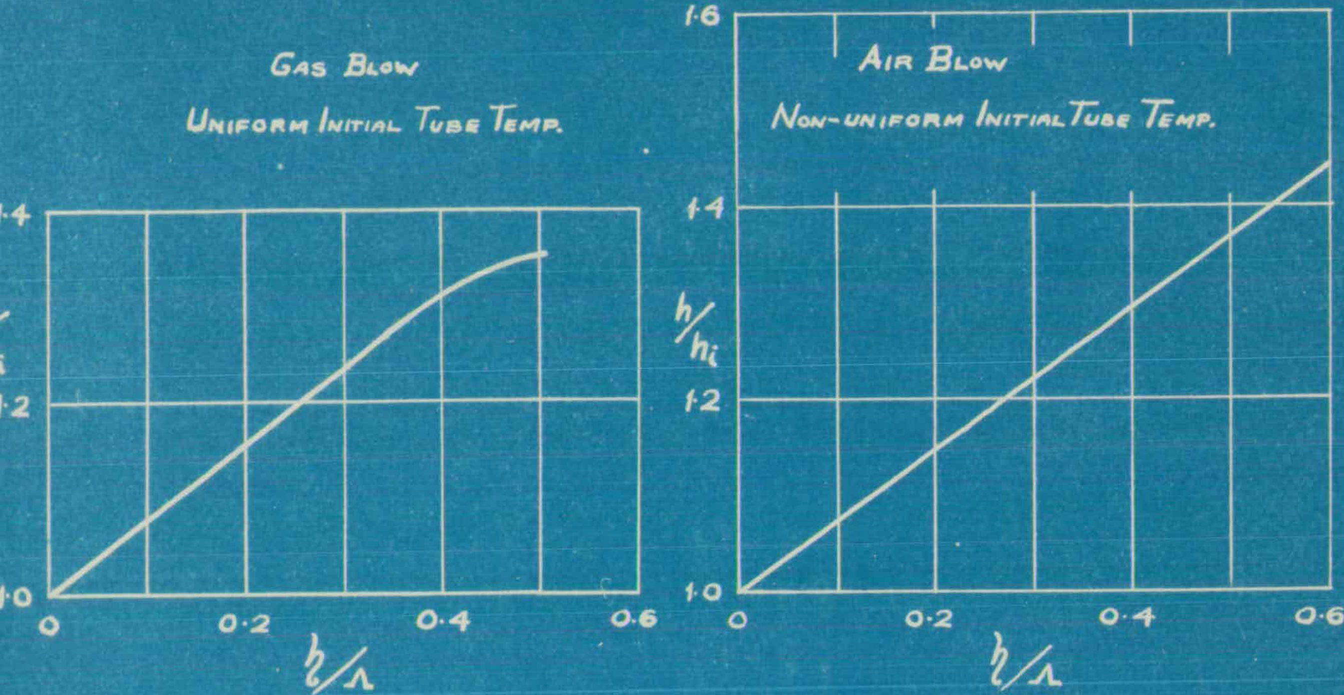


FIG. 18 a.

UNIFORM INITIAL TUBE TEMPERATURE

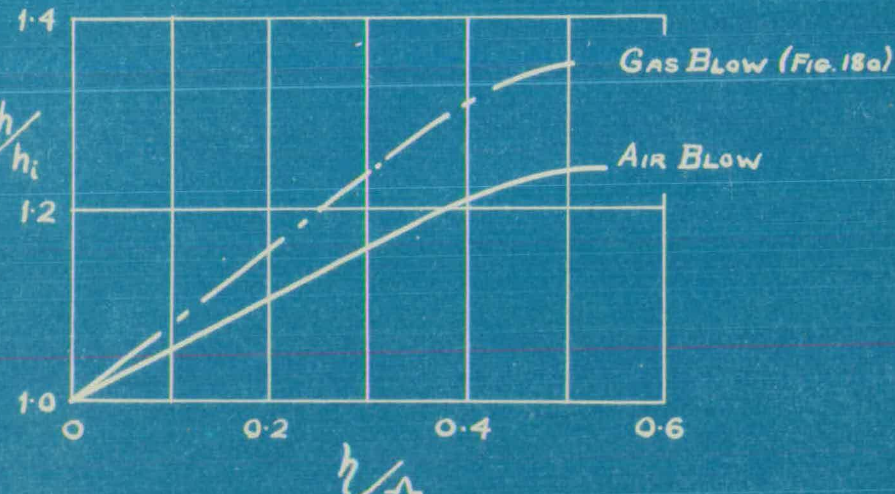


FIG. 18 b.

TIME VARIATION OF HEAT TRANSFER COEFFICIENT.

AVERAGE CURVES DERIVED FROM ALL SINGLE BLOW RESULTS

h_i = VALUE AT CHANGEOVER i.e. $\eta = 0$.

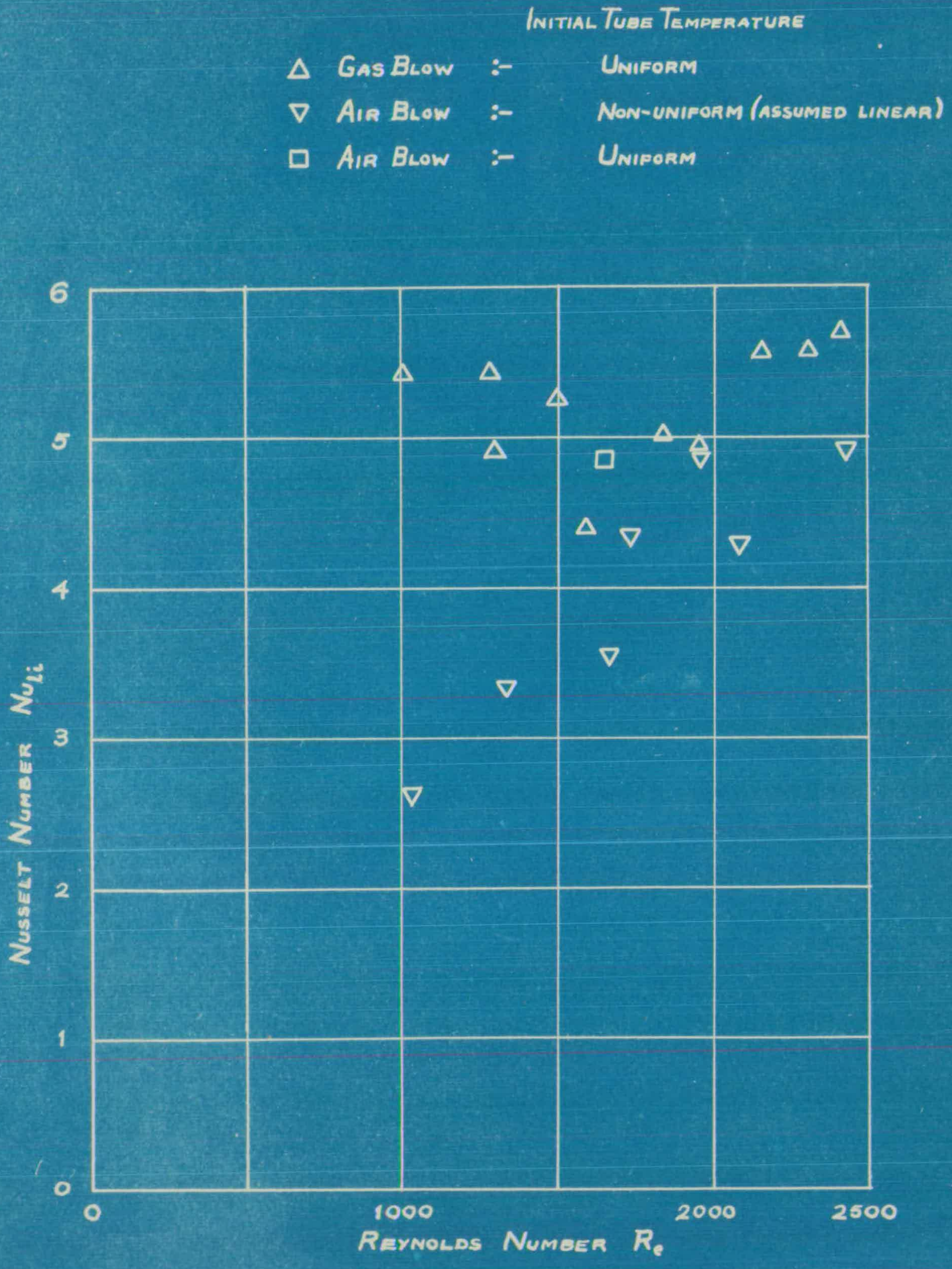


FIG. 19. NUSSELT NUMBER VALUES.

Single Blow Tests , $\eta = 0$.

Values obtained assuming Uniform Supply Inlet Temperature

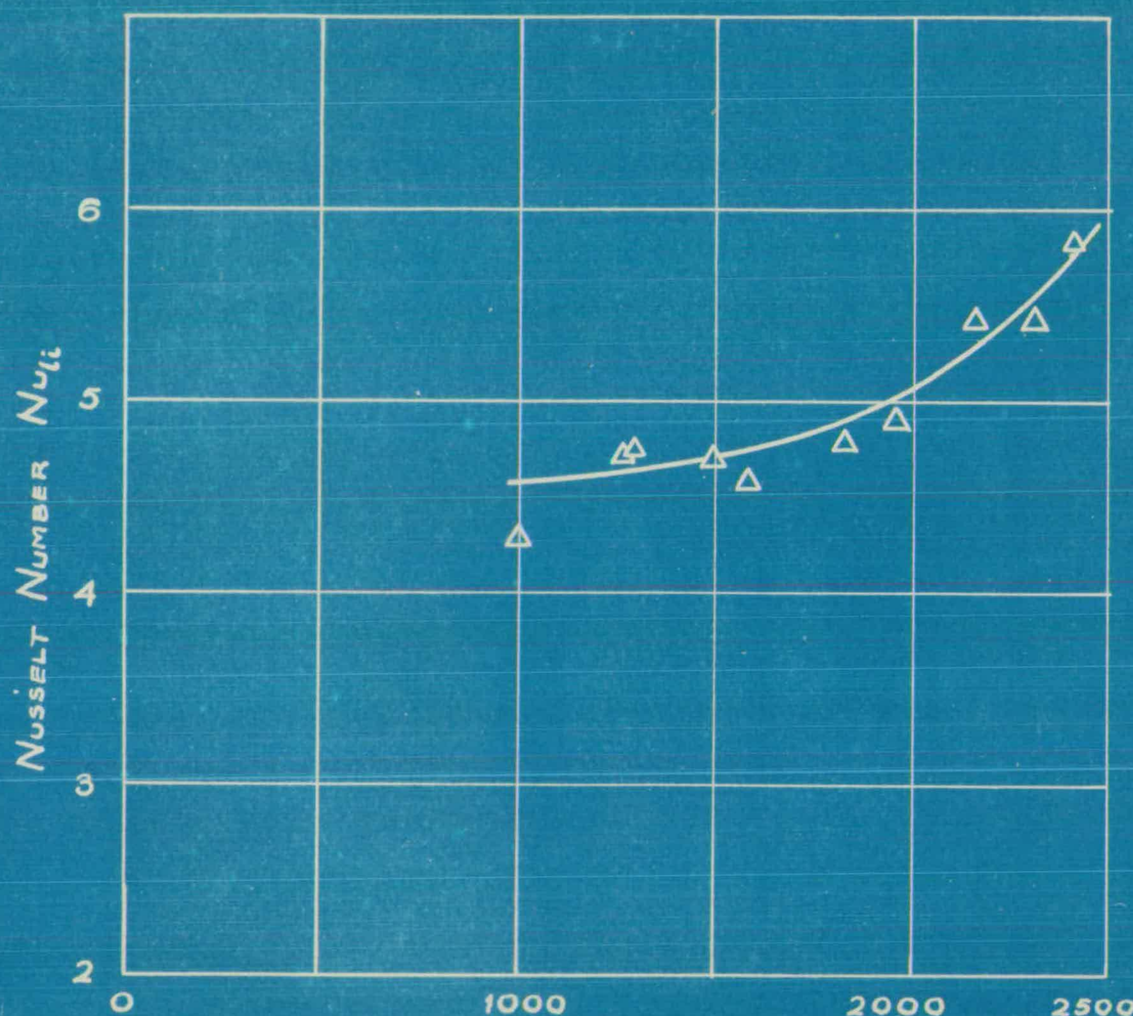


FIG. 20. NUSSELT NUMBER VALUES.

Single Gas Blow (Hot Supply), $\eta = 0$.

Values obtained for Actual Test Boundary Conditions

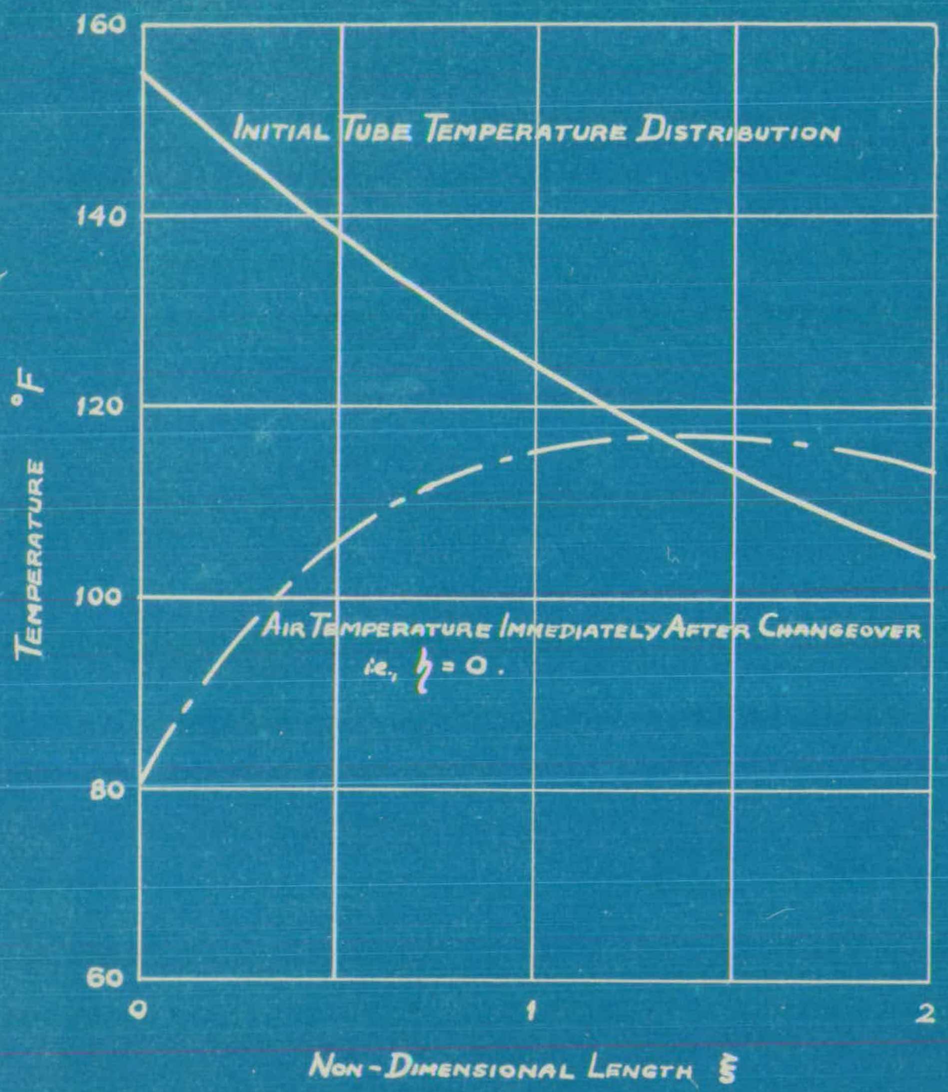


FIG. 21. TUBE AND AIR TEMPERATURES.

SINGLE AIR BLOW, $\eta = 0$.

CALCULATED VALUES

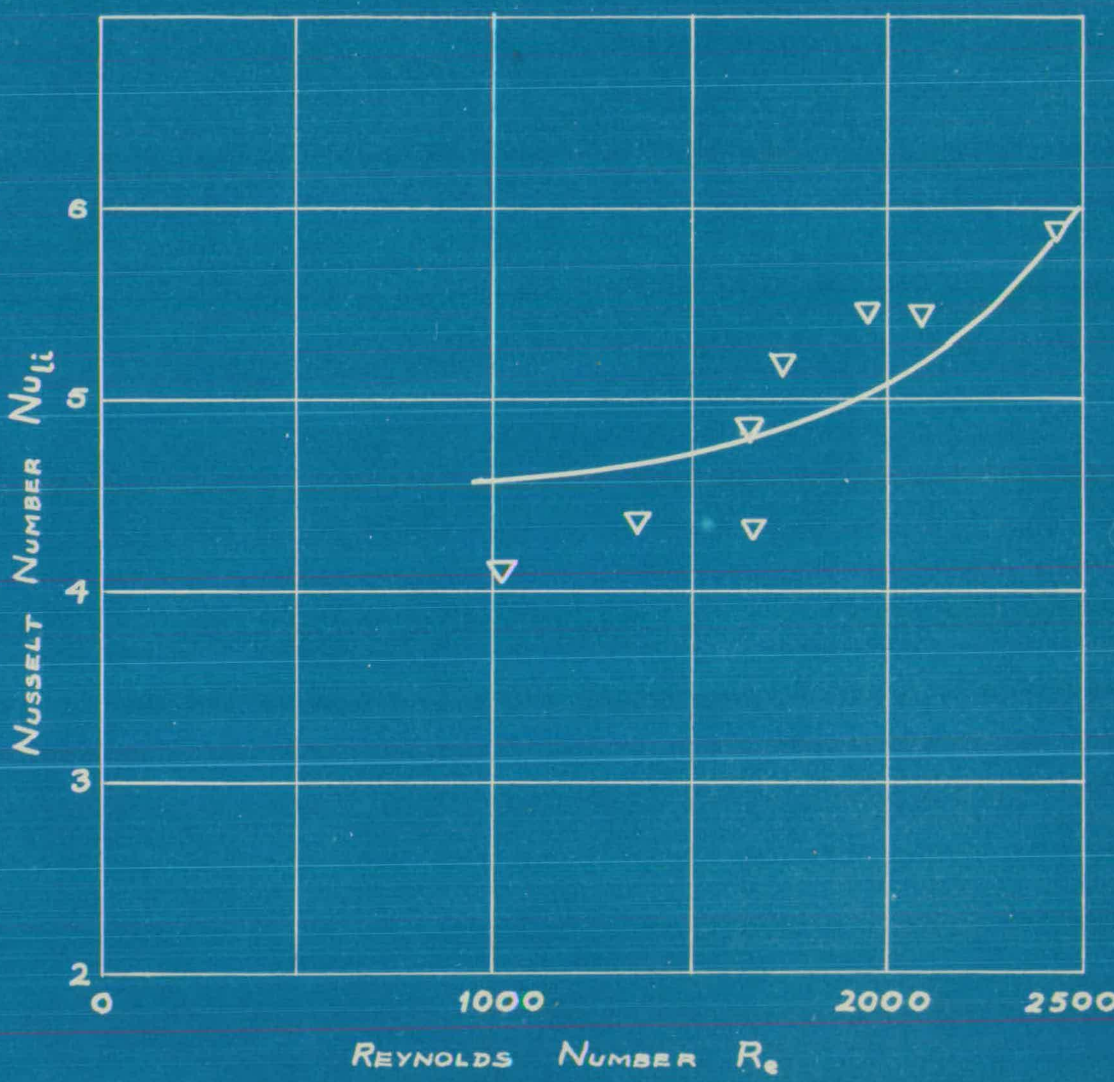


FIG. 22. NUSSELT NUMBER VALUES.

Single Air Blow (Cold Supply), $\eta = 0$.

Values obtained for Actual Test Boundary Conditions

Uniform Initial Tube Temperature

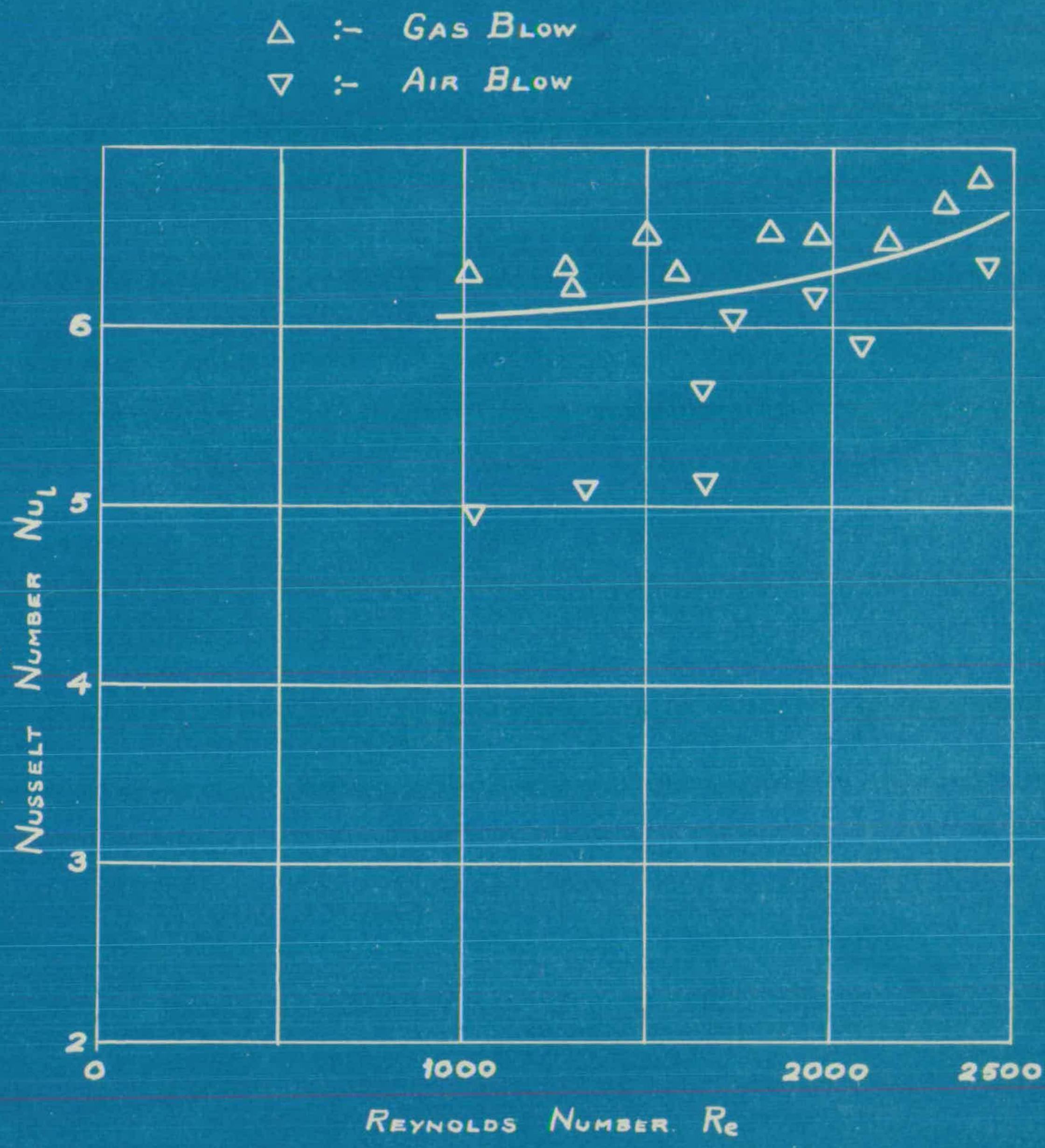


FIG. 23. NUSSELT NUMBER VALUES.

SINGLE BLOW – GAS AND AIR, $h/\Lambda = 0{\cdot}3$.

VALUES OBTAINED FOR ACTUAL TEST BOUNDARY CONDITIONS

UNIFORM INITIAL TUBE TEMPERATURE

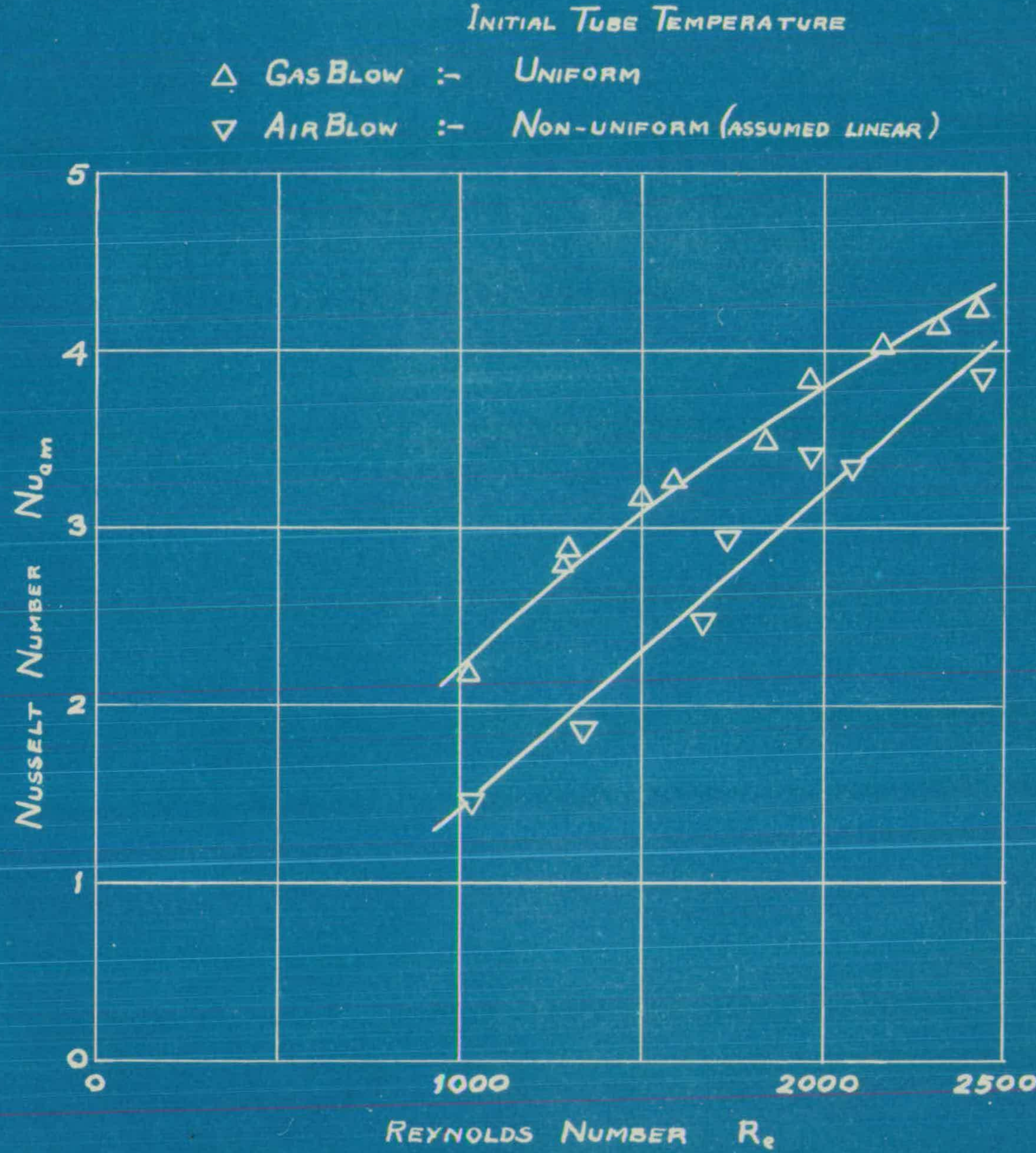


FIG. 24. NUSSELT NUMBER VALUES.

BASED ON ARITHMETIC MEAN TEMPERATURE DIFFERENCE

SINGLE BLOW TESTS, $\lambda = 0$.

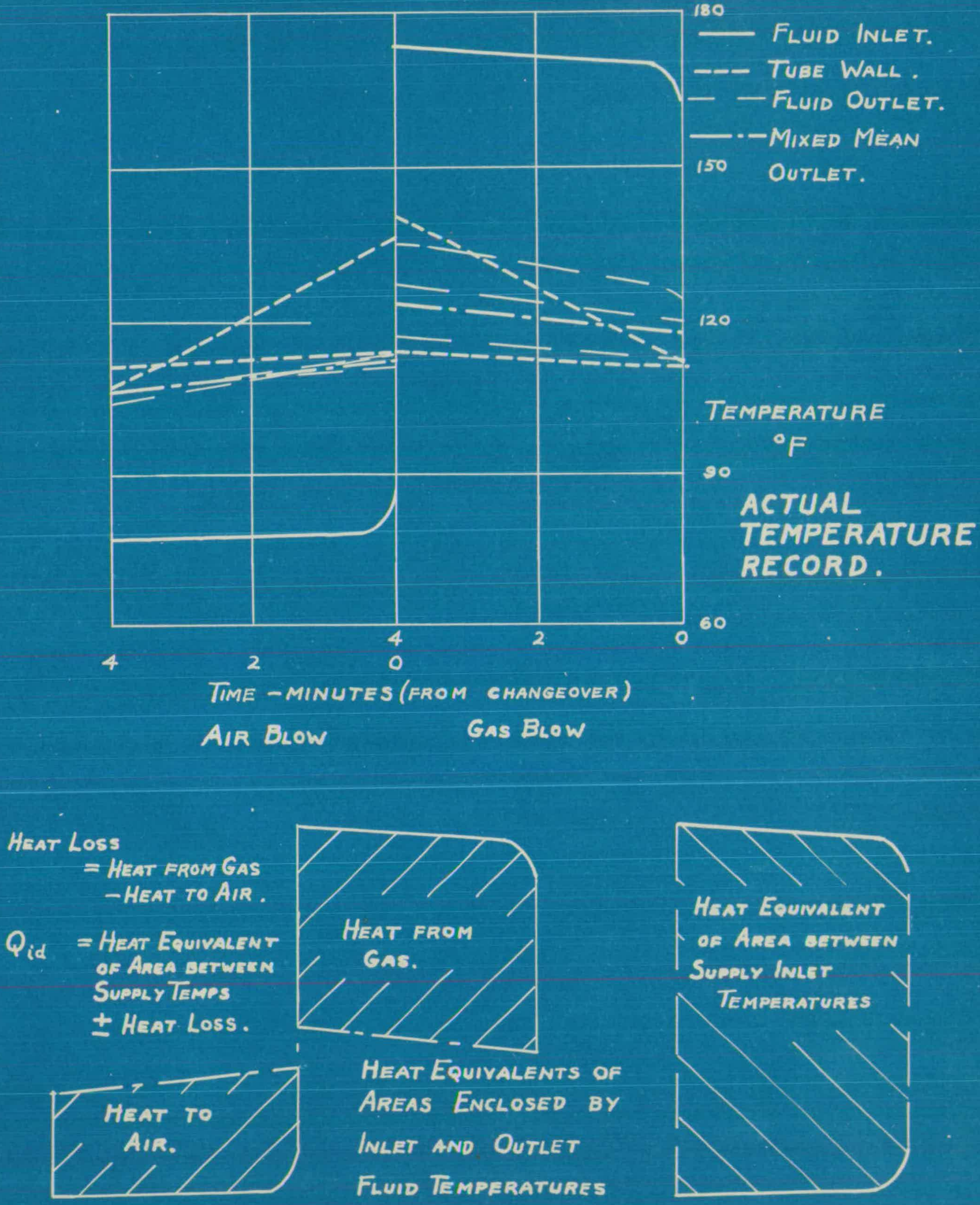


FIG. 25. CYCLIC OPERATION.